The density theorem for the Muchnik lattice

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Let $\mathcal{E}_{\mathrm{T}}$ be the upper semilattice of recursively enumerable Turing degrees. There is a strong analogy between $\mathcal{E}_{\mathrm{T}}$ and $\mathcal{E}_{\mathrm{w}}$, the lattice of Muchnik degrees of nonempty Π^0_1 subsets of $\{0,1\}^{\mathbb{N}}$. The most famous structural theorems concerning $\mathcal{E}_{\mathrm{T}}$ are the Splitting Theorem (Sacks 1962) and the Density Theorem (Sacks 1964). The analogous Splitting Theorem for $\mathcal{E}_{\mathrm{w}}$ was proved by Stephen Binns and published in 2003. The analogous Density Theorem for $\mathcal{E}_{\mathrm{w}}$ has been a long-standing open question, but it was recently proved by Binns, Shore, and Simpson and will appear in a forthcoming paper. In this talk I shall sketch the proof of the Density Theorem for $\mathcal{E}_{\mathrm{w}}$. The proof involves a small amount of hyperarithmetical theory.