Well partial orderings and better partial orderings, with applications to algebra

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February 4, 2016

This is my abstract for a workshop on Ordered Algebras and Logic, to be held February 6–7, 2016 at Vanderbilt University.

A well partial ordering is a partial ordering with no infinite descending sequences and no infinite antichains. This concept arises frequently in algebra. For example, a standard proof of the Hilbert Basis Theorem uses Dickson's Lemma: for each positive integer k, the set of k-tuples of natural numbers with the product ordering is a well partial ordering. As another application, there is a theorem of Formanek and Lawrence: the group ring of the infinite symmetric group is Noetherian, i.e., it has no infinite ascending chain of two-sided ideals. The class of well partial orderings has certain finitary closure properties. The better partial orderings are a subclass of the well partial orderings, but with analogous infinitary closure properties. The concept of better partial orderings was introduced by Nash-Williams and used by Laver to prove a conjecture of Fraisse: the class of countable linear orderings is well partially ordered under embeddability. As an algebraic application of better partial orderings, we now prove that the group ring of the infinite symmetric group has the antichain condition, i.e., it has no infinite antichain of two-sided ideals.