

# Degrees of unsolvability of 2-dimensional subshifts of finite type

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## Abstract

We apply some fundamental concepts and results from mathematical logic in order to obtain an apparently new counterexample in symbolic dynamics. Two sets  $X$  and  $Y$  are said to be *strongly equivalent* if there exist partial recursive functionals from  $X$  into  $Y$  and vice versa. The *strong degree* of  $X$  is the equivalence class of  $X$  under strong equivalence. There is an extensive recursion-theoretic literature on the lattice of strong degrees of nonempty  $\Pi_1^0$  subsets of the Cantor space. This lattice is known as  $\mathcal{P}_s$ . We prove that  $\mathcal{P}_s$  consists precisely of the strong degrees of 2-dimensional subshifts of finite type. We use this result to obtain an infinite collection of 2-dimensional subshifts of finite type which are, in a certain sense, mutually incompatible.

## References

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