Reverse mathematics and the ACC

Stephen G. Simpson
Department of Mathematics
Vanderbilt University, USA
http://www.math.psu.edu/simpson/
sgslogic@gmail.com

September 10, 2019

This is my abstract for a talk at the workshop *Reverse Mathematics of Combinatorial Principles*, to be held at BIRS-CMO in Oaxaca, Mexico, September 15–20, 2019.

Reverse mathematics seeks to classify core-mathematical theorems up to logical equivalence. In this talk I discuss some results to the effect that certain algebraic theorems are reverse-mathematically equivalent to the well-orderedness of certain ordinal numbers. A long time ago I showed that the Hilbert Basis Theorem (1890: "every ideal in a polynomial ring is finitely generated") is equivalent to well-orderedness of ω^ω, hence not finitistically reducible in the sense of Hilbert's Program in the foundations of mathematics. This reverse-mathematical result supports Gordan's famous remark to the effect that the Hilbert Basis Theorem is not mathematics but theology. Some related theorems due to Robson and MacLagen are known to be even more theological, in that they are equivalent to well-orderedness of ω^{ω^ω}. Let $K[S]$ denote the group ring of the infinite symmetric group S. A 1976 theorem of Formanek and Lawrence says that the 2-sided ideals of $K[S]$ satisfy the ascending chain condition. Recently Hatzikiriakou and I used b.q.o. theory to show that these same ideals satisfy the antichain condition. For these ideals we show that the ascending chain condition is equivalent to well-orderedness of ω^ω, but the status of the antichain condition remains as an open question.