Reverse Mathematics and Π^1_2 Comprehension

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Abstract

This is joint work with Carl Mummert. We initiate the reverse mathematics of general topology. We show that a certain metrization theorem is equivalent to Π^1_2 comprehension. If P is a poset, let $\mathrm{MF}(P)$ be the space of maximal filters on P. Here $\mathrm{MF}(P)$ has the obvious topology generated by basic open sets $N_p = \{F \in \mathrm{MF}(P) \mid p \in F\}$, $p \in P$. An MF space is defined to be a topological space of the form $\mathrm{MF}(P)$. If P is countable, we say that $\mathrm{MF}(P)$ is countably based. The class of countably based MF spaces can be defined and discussed within the subsystem ACA_0 of second-order arithmetic. One can prove within ACA_0 that every complete separable metric space is regular and is homeomorphic to a countably based MF space. We show that the converse statement, "every regular, countably based MF space is homeomorphic to a complete separable metric space," is equivalent to Π^1_2-CA_0. The equivalence is proved in the weaker system Π^1_1-CA_0. This is the first example of a theorem of core mathematics which is provable in second-order arithmetic and implies Π^1_2 comprehension.