Computable symbolic dynamics

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This is the abstract of my talk at a workshop on algorithmic randomness at the University of Wisconsin, May 27–31, 2009.

Abstract

Let G be a computable group, and let A be a finite alphabet. By a *G-subshift* we mean a nonempty subset of A^G which is topologically closed and closed under the action of G. It can be shown that any G-subshift X is defined by a countable set E of excluded finite configurations. If E is finite, we say that X is *of finite type*. If E is computable, we say that X is *of computable type*. It can be shown that most or all G-subshifts which arise in practice are of computable type. Let X be of computable type, and let $P(X)$ be the problem of finding a point of X. If X is *minimal* (i.e., every orbit is dense) and of computable type, then $P(X)$ is algorithmically solvable (a result of Michael Hochman). Let us say that X is *d-dimensional* if $G = \mathbb{Z}^d$. If X is 1-dimensional and of computable type, then $P(X)$ can be of any desired degree of unsolvability (a result of Joseph Miller). If X is 2-dimensional and of finite type, then $P(X)$ can be of any desired degree of unsolvability (my result). The 1-dimensional subshifts of computable type are precisely those which can be obtained as projections of 2-dimensional subshifts of finite type (a result of Alexander Shen).