

Predicativity: The Outer Limits

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Abstract

Beginning with ideas of Poincaré and Weyl, Feferman in the sixties undertook a profound analysis of the predicativist foundational program. He presented a subsystem of second order arithmetic IR and argued convincingly that it represents the outer limits of what is predicatively provable. Much later, Friedman introduced another system ATR_0 which is conservative over IR for Π_1^1 sentences yet includes several well known theorems of algebra, descriptive set theory, and countable combinatorics that are not provable in IR . The proof-theoretic ordinal of both systems is Γ_0 . ATR_0 has emerged as one of a handful of systems that are important for reverse mathematics. From a foundational standpoint, we may say that IR represents predicative provability while ATR_0 represents predicative reducibility. Subsequently Friedman formulated mathematically natural finite combinatorial theorems that are not only not predicatively provable but go beyond Γ_0 and therefore are not predicatively reducible. I plan to report on recent developments in this area.