Degrees of unsolvability: a two-hour tutorial

Stephen G. Simpson
Department of Mathematics, Pennsylvania State University
Department of Mathematics, Vanderbilt University
http://www.math.psu.edu/simpson/
stephen.g.simpson@vanderbilt.edu
sgslogic@gmail.com

September 23, 2016

This is my abstract for Computability Theory and Foundations of Mathematics (CTFM 2016), Waseda University and Tokyo Institute of Technology, September 20–23, 2016.

Given a problem P, one associates to P a *degree of unsolvability*, denoted $\deg(P)$. Here $\deg(P)$ is a quantity which measures the "difficulty" of P, i.e., the amount of algorithmic unsolvability which is inherent in P. We focus on two degree structures: the semilattice of Turing degrees, $\mathcal{D}_{\mathrm{T}}$, and its completion, $\mathcal{D}_{\mathrm{w}} = \widehat{\mathcal{D}_{\mathrm{T}}}$, the lattice of Muchnik degrees. We remark that $\mathcal{D}_{\mathrm{w}}$ gives rise to a natural recursion-theoretic interpretation of Kolmogorov's non-rigorous "calculus of problems." We emphasize the analogy between $\mathcal{E}_{\mathrm{T}}$, the countable subsemilattice of $\mathcal{D}_{\mathrm{T}}$ consisting of the Turing degrees associated with recursively enumerable subsets of $\mathbb{N}$, and $\mathcal{E}_{\mathrm{w}}$, the countable sublattice of $\mathcal{D}_{\mathrm{w}}$ consisting of the Muchnik degrees associated with nonempty Π^0_1 subsets of $\{0,1\}^{\mathbb{N}}$. We emphasize specific, natural degrees in $\mathcal{D}_{\mathrm{T}}$, $\mathcal{D}_{\mathrm{w}}$, and $\mathcal{E}_{\mathrm{w}}$.