Reverse mathematics and the ACC

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January 24, 2014

This is my abstract for Computability Theory and Foundations of Mathematics (CTFM 2014), Tokyo Institute of Technology, February 17–20, 2014.

In abstract algebra, a ring is said to satisfy the ACC (ascending chain condition) if it has no infinite ascending sequence of ideals. A famous theorem of Hilbert, 1890, says that polynomial rings with finitely many indeterminates satisfy the ACC. There is also a similar theorem for noncommuting indeterminates, due to J. C. Robson, 1978. In 1988 I performed a reverse-mathematical analysis of the theorems of Hilbert and Robson, proving that they are equivalent over RCA_0 to the well-orderedness of ω^ω and ω^{ω^ω} respectively. Now I perform a similar analysis of a theorem of E. Formanek and J. Lawrence, 1976. Let S be the group of finitely supported permutations of the natural numbers. Let $K[S]$ be the group ring of S over a countable field K of characteristic 0. Formanek and Lawrence proved that $K[S]$ satisfies the ACC. I now prove that the Formanek/Lawrence theorem is equivalent over RCA_0 to the well-orderedness of ω^ω. I also show that, in all of these reverse-mathematical results, RCA_0 can be weakened to RCA_0^*. This recent work was done jointly with Kostas Hatzikiriakou.

In addition, I make some remarks concerning reverse mathematics as it applies to Hilbert's foundational program of finitistic reductionism. It is significant that RCA_0 and WKL_0 and even $\mathsf{WKL}_0 + \Sigma^0_2$-bounding are conservative for Π^0_2 sentences over PRA, while Σ_2-induction and the well-orderedness of ω^ω are not.