# Propagation of partial randomness

Stephen G. Simpson
Department of Mathematics
Pennsylvania State University
http://www.math.psu.edu/simpson/

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Let $x$ be an infinite sequence of 0's and 1's, i.e., $x \in \{0,1\}^{\mathbb{N}}$. Even if $x$ is not Martin-Löf random, we can nevertheless quantify the amount of *partial randomness* which is inherent in $x$. Several researchers including Tadaki have studied partial randomness. We now present some new results due to Higuchi, Hudelson, Simpson and Yokoyama concerning propagation of partial randomness. Our results say that if $x$ has a specific amount of partial randomness, then $x$ has an equal amount of partial randomness relative to certain Turing oracles. Let KA denote *a priori* Kolmogorov complexity, i.e., $\mathrm{KA}(\sigma) = -\log_2 m(\sigma)$ where $m$ is Levin's universal left-r.e. semimeasure. Note that KA is similar but not identical to the more familiar KP = *prefix-free* Kolmogorov complexity. Given a computable function $f : \{0,1\}^* \to (-\infty, \infty)$, we say that $x \in \{0,1\}^{\mathbb{N}}$ is *strongly $f$-random* if $\exists c \, \forall n \, (\mathrm{KA}(x{\restriction}\{1,\ldots,n\}) > f(x{\restriction}\{1,\ldots,n\}) - c)$. Two of our results read as follows. Theorem 1. Assume that $x$ is strongly $f$-random and Turing reducible to $y$ where $y$ is Martin-Löf random relative to $z$. Then $x$ is strongly $f$-random relative to $z$. Theorem 2. Assume that $\forall n \, (x_n$ is strongly $f_n$-random). Then, we can find a PA-oracle $z$ such that $\forall n \, (x_n$ is strongly $f_n$-random relative to $z$). We also show that Theorems 1 and 2 fail with KA replaced by KP.