Reverse mathematics and the ACC

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This is my abstract for a logic seminar talk at Cornell University, October 19–22, 2015.

Reverse mathematics seeks to classify core-mathematical theorems up to logical equivalence. In this talk we discuss some results to the effect that certain algebraic theorems are reverse-mathematically equivalent to the well-orderedness of certain ordinal numbers. Let ω_n denote the ordinal number consisting of a stack of omegas of height n. A long time ago I showed that the Hilbert Basis Theorem (1890: "every ideal in a polynomial ring is finitely generated") is equivalent to well-orderedness of ω_2, hence not finitistically reducible in the sense of Hilbert's Program in the foundations of mathematics. This reverse-mathematical result supports Gordan's famous remark to the effect that the Hilbert Basis Theorem is not mathematics but theology. Some related theorems due to Robson and MacLagen are known to be even more theological, in that they are equivalent to well-orderedness of ω_3. Let $K[S]$ denote the group ring of the infinite symmetric group S. A 1976 theorem of Formanek and Lawrence says that the 2-sided ideals of $K[S]$ satisfy the ascending chain condition. Recently Hatzikiriakou and I used b.q.o. theory to show that these same ideals satisfy the antichain condition. For these ideals we show that the ascending chain condition is equivalent to well-orderedness of ω_2, but the status of the antichain condition remains as an open question.