# Propagation of partial randomness

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Let $X$ be an infinite sequence of 0's and 1's. Even if $X$ is not Martin-Löf random, we can nevertheless quantify the amount of *partial randomness* which is inherent in $X$. Many researchers including Calude, Hudelson, Kjos-Hanssen, Merkle, Miller, Reimann, Staiger, Tadaki, and Terwijn have studied partial randomness. We now present some new results due to Higuchi, Hudelson, Simpson and Yokoyama concerning *propagation of partial randomness*. Our results say that if $X$ has a specific amount of partial randomness, then $X$ has an equal amount of partial randomness relative to certain Turing oracles. To be precise, let KA denote a priori Kolmogorov complexity, i.e., $\mathrm{KA}(\sigma) = -\log_2 m(\sigma)$ where $m$ is Levin's universal left-r.e. semimeasure. Note that KA is similar but not identical to the more familiar KP = prefix-free Kolmogorov complexity. Let $\mathbb{N} = \{1, 2, 3, \ldots\}$. Given a computable function $f : \{0,1\}^* \to \mathbb{N}$, we say that $X \in \{0,1\}^{\mathbb{N}}$ is strongly $f$-random if $\exists c \, \forall n \, (\mathrm{KA}(X{\upharpoonright}\{1,\ldots,n\}) > f(X{\upharpoonright}\{1,\ldots,n\}) - c)$. We show that $X$ is autocomplex in the sense of Kjos-Hanssen/Merkle/Stephan if and only if $X$ is strongly $f$-random for some $f$ such that $\{f(X{\upharpoonright}\{1,\ldots,n\}) \mid n \in \mathbb{N}\}$ is unbounded. Thus the concept of strong $f$-randomness provides a fine hierarchy of autocomplexity. Two of our propagation results read as follows. Theorem 1. Assume that $X$ is strongly $f$-random and Turing reducible to $Y$ where $Y$ is Martin-Löf random relative to $Z$. Then $X$ is strongly $f$-random relative to $Z$. Theorem 2. Assume that $\forall i \, (X_i$ is strongly $f_i$-random). Then, we can find a PA-oracle $Z$ such that $\forall i \, (X_i$ is strongly $f_i$-random relative to $Z$). We also show that Theorems 1 and 2 fail if KA is replaced by KP.