Symbolic dynamics: entropy = dimension = complexity

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December 14, 2010

This is my abstract for an invited talk at the conference Computability and Complexity in Analysis 2011, to be held at the University of Cape Town, South Africa, January 31 to February 4, 2011.

We prove some fundamental results in symbolic dynamics. Let G be a countable semigroup, specifically $G = (\mathbb{N}^d, +)$ or $G = (\mathbb{Z}^d, +)$ where d is a positive integer. Let A be a finite set of symbols. A *G-subshift* is a nonempty set $X \subseteq A^G$ which is G-invariant and topologically closed. Note that we impose no finiteness or computability hypothesis on X. We show that, with respect to the standard metric on X, the Hausdorff dimension of X coincides with the effective Hausdorff dimension of X and with the topological entropy of X. We obtain a sharp characterization of the Hausdorff dimension of X in terms of the Kolmogorov complexity of the finite configurations of symbols which occur in X.