The Gödel Hierarchy and Reverse Mathematics

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Abstract

The Gödel Hierarchy is an array of foundationally significant theories in the predicate calculus. The theories range from weak (bounded arithmetic, elementary function arithmetic) through intermediate (subsystems of second-order arithmetic), through strong (Zermelo/Fraenkel set theory, large cardinals). The theories are ordered by inclusion, interpretability, and consistency strength. Reverse Mathematics is a program which seeks to classify mathematical theorems by calibrating their places within the Gödel Hierarchy. The theorems are drawn from core mathematical areas such as analysis, algebra, functional analysis, topology, and combinatorics. Remarkably, the Reverse Mathematics classification scheme exhibits a considerable amount of regularity and structure. In particular, a large number of core mathematical theorems fall into a small number of foundationally significant equivalence classes (the so-called "big five"). There are close connections with other foundational programs and hierarchies. In particular, concepts and methods from degrees of unsolvability play an important role.