Symbolic dynamics and degrees of unsolvability

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Abstract

Let A be a finite set of symbols. The 2-*dimensional shift space on* A is $A^{\mathbb{Z}\times\mathbb{Z}}$ with shift operators S_1 and S_2 given by $S_1(x)(m,n) = x(m+1,n)$ and $S_2(x)(m,n) = x(m,n+1)$. A 2-*dimensional subshift* is a nonempty, closed subset of $A^{\mathbb{Z}\times\mathbb{Z}}$ which is invariant under S_1 and S_2. A 2-dimensional subshift is said to be *of finite type* if it is defined by a finite set of excluded finite configurations of symbols. We regard real numbers and points of $A^{\mathbb{Z}\times\mathbb{Z}}$ as Turing oracles. If X and Y are sets of Turing oracles, we say that X *is Muchnik reducible to* Y if each $y \in Y$ can be used to compute some $x \in X$. The *Muchnik degree of* X is the equivalence class of X under mutual Muchnik reducibility. We prove that the Muchnik degrees of 2-dimensional subshifts of finite type are the same as the Muchnik degrees of nonempty, effectively closed sets of real numbers. We then apply known results about such Muchnik degrees to obtain an infinite family of 2-dimensional subshifts of finite type which are, in a certain strong sense, mutually independent. Our application is stated purely in terms of symbolic dynamics, with no mention of Muchnik reducibility.