

REVERSE MATHEMATICS

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Abstract. Foundations of mathematics is the study of the most basic concepts and logical structure of mathematics as a whole. Reverse mathematics is the foundational program of discovering which set existence axioms are needed to prove known theorems in core mathematical areas such as algebra, analysis, geometry, countable combinatorics. It turns out that a large number of theorems fall into a small number of equivalence classes with respect to provable equivalence over a weak base theory. (See my book *Subsystems of Second Order Arithmetic*, Springer-Verlag, 1999.) The equivalence classes appear to reflect well known philosophical/foundational programs such as constructivism (Bishop), finitistic reductionism (Hilbert), predicativism (Weyl, Feferman), predicative reductionism, and impredicative analysis. In this talk I comment on the mathematical, philosophical, and foundational significance of reverse mathematics as it has developed from the early 1970's to the present. I also comment on the significance of reverse mathematics for the principal subdivisions of mathematical logic: model theory, set theory, recursion theory, proof theory.

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