

Mass Problems and Hyperarithmeticity

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Abstract

A mass problem is a set of Turing oracles, viewed as a “decision problem with more than one solution”. A mass problem P is said to be weakly reducible to a mass problem Q if for each $Y \in Q$ there exists $X \in P$ such that $X \leq_T Y$. Beginning in 1999 Simpson and others studied the partial ordering \mathcal{P}_w of mass problems associated with nonempty Π_1^0 subsets of 2^ω under weak reducibility. One easily sees that \mathcal{P}_w is a countable distributive lattice with 0 and 1. Simpson found a natural embedding of the recursively enumerable Turing degrees into \mathcal{P}_w preserving \leq , \vee , 0, 1. In addition, Simpson showed that \mathcal{P}_w contains many specific, natural degrees other than the recursively enumerable Turing degrees. These specific, natural degrees are related to foundationally interesting topics such as reverse mathematics, algorithmic randomness, subrecursive hierarchies, and computational complexity. Recently Simpson exhibited a new family of specific, natural degrees in \mathcal{P}_w related to hyperarithmetical theory. Recently Simpson in joint work with Josh Cole showed that certain index sets associated with weak reducibility in \mathcal{P}_w are Π_1^1 complete.