

Reverse Topology

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Abstract

Until now there has been no Reverse Mathematics study of general topology. The main obstacle has been that there is no way to express propositions concerning arbitrary topological spaces, even separable or second countable ones, in the language of second order arithmetic. We now extend Reverse Mathematics into the realm of general topology, but the extension is limited to a class of topological spaces called *second countable poset spaces* which can be encoded nicely as countable objects. We show that every complete separable metric space is a second countable poset space. We conjecture that the Urysohn Metrization Theorem for second countable poset spaces is equivalent over ACA_0 to Π_1^1 comprehension. One of our results is that ATR_0 is equivalent over ACA_0 to the assertion that every second countable poset space has either countably or continuum many points. This is joint work with Carl Mummert.