Mass Problems to the Rescue!

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Abstract

Let $\mathcal{R}_T$ be the countable upper semilattice of recursively enumerable Turing degrees. Post's 1944 paper suggested two profound problems concerning $\mathcal{R}_T$, both of which remain unsolved. (1) To find a specific, natural example of a recursively enumerable Turing degree other than $0'$ and 0. (2) To find a "smallness property" of the complement of a coinfinite recursively enumerable set A which ensures that the Turing degree of A is $< 0'$. The purpose of this talk is to point out that, by moving to a slightly wider context, one obtains satisfactory solutions to problems (1) and (2). Instead of $\mathcal{R}_T$, consider the countable distributive lattice $\mathcal{P}_w$ of weak degrees, a.k.a., Muchnik degrees, of mass problems associated with nonempty Π^0_1 subsets of 2^ω. It is known that $\mathcal{R}_T$ is embedded in $\mathcal{P}_w$ via a natural embedding which carries $0'$ and 0 to the top and bottom elements of $\mathcal{P}_w$, respectively. We identify $\mathcal{R}_T$ with its image in $\mathcal{P}_w$ under this embedding. Regarding problem (1), there are a variety of specific, natural degrees in $\mathcal{P}_w$ which are related to foundationally interesting topics such as reverse mathematics, algorithmic randomness, subrecursive hierarchies, and computational complexity. Unfortunately, all of these specific, natural degrees in $\mathcal{P}_w$ turn out to be incomparable with all of the recursively enumerable Turing degrees, except $0'$ and 0. Regarding problem (2), it turns out that there are "smallness properties" of nonempty Π^0_1 sets $P \subseteq 2^\omega$, analogous to simplicity, hypersimplicity, and hyperhypersimplicity, each of which implies that the weak degree of P is $< 0'$. (Some of this work is joint with Stephen Binns.)