Stephen G. Simpson (`simpson@math.psu.edu`), Department of Mathematics, Pennsylvania State University, University Park, State College PA 16802. *Muchnik Degrees of* Π^0_1 *Subsets of* 2^ω*.*

Let $\mathcal{P}$ be the set of nonempty Π^0_1 subsets of 2^ω. For $P, Q \in \mathcal{P}$ say that P is *Muchnik reducible* to Q, abbreviated $P \leq_w Q$, if for all $Y \in Q$ there exists $X \in P$ such that X is Turing reducible to Y. The *Muchnik degree* of P is the equivalence class of P under the equivalence relation $P \equiv_w Q$, i.e., $P \leq_w Q$ and $Q \leq_w P$. The Muchnik degrees of members of $\mathcal{P}$ form a countable distributive lattice with 0 and 1. Call this lattice $\mathcal{P}_w$. By Binns/Simpson 2000, every countable distributive lattice is lattice embeddable in every nontrivial initial segment of $\mathcal{P}_w$. We prove that $P \in \mathcal{P}$ is of Muchnik degree 1 if and only if the Turing degrees of members of P are precisely the Turing degrees of completions of Peano Arithmetic. We prove that, among Muchnik degrees of members of $\mathcal{P}$ of positive measure, there is a largest one, call it $\mathbf{r}_1$. We prove that $\mathbf{r}_1 \in \mathcal{P}_w$ is meet-irreducible and does not join to 1. Call $P \in \mathcal{P}$ *thin* if $P \setminus Q \in \mathcal{P}$ for all $Q \in \mathcal{P}$. By Martin/Pour-El 1970, there exists a thin $P \in \mathcal{P}$ such that the Muchnik degree of P is not 0. By Downey 1982/1987 and Downey/Jockusch/Stob 1990/1996, many such Muchnik degrees exist. We prove that all such Muchnik degree are incomparable with $\mathbf{r}_1$.