

# Math 558 – Homework #3

Due October 29, 2009

1. Find a pair of numbers  $r, a$  such that  $\beta(r, a, 0) = 11$ ,  $\beta(r, a, 1) = 19$ ,  $\beta(r, a, 2) = 30$ ,  $\beta(r, a, 3) = 37$ ,  $\beta(r, a, 4) = 51$ .

Hint: First find an appropriate  $a$  by hand. Then write a small computer program to find  $r$  by brute force.

2. Recall that  $\mathbb{N} = \{0, 1, 2, \dots\}$  = the natural numbers,  
 $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$  = the integers, and  
 $\mathbb{R} = (-\infty, \infty)$  = the real numbers.

According to Matiyasevich's Theorem, we can find a polynomial

$$f(w, x_1, \dots, x_k)$$

with integer coefficients, such that the set of  $a \in \mathbb{N}$  for which the equation  $f(a, x_1, \dots, x_k) = 0$  has a solution in  $\mathbb{N}$  is noncomputable.

- (a) Discuss the analogous question in which “solution in  $\mathbb{N}$ ” is replaced by “solution in  $\mathbb{Z}$ ”.
  - (b) Discuss analogous questions in which “solution in  $\mathbb{N}$ ” is replaced by “solution in  $\mathbb{R}$ ”.
3. Prove König's Theorem:

Let  $\langle \kappa_i \rangle_{i \in I}$  and  $\langle \lambda_i \rangle_{i \in I}$  be indexed sets of cardinal numbers with the same index set  $I$ . If  $\kappa_i < \lambda_i$  for all  $i \in I$ , then  $\sum_{i \in I} \kappa_i < \prod_{i \in I} \lambda_i$ .