# Math 558 – Homework #1

Due September 15, 2009

1. A real number $\alpha$ is said to be *primitive recursive* if the function $f(n) =$ the $n$th digit of $\alpha$ is primitive recursive. A real number $\alpha$ is said to be *algebraic* if it is a root of a nonzero polynomial with integer coefficients. For example, $\sqrt{2}$ is a real algebraic number, because it is a root of the polynomial $x^2 - 2$.

   Prove that all real algebraic numbers are primitive recursive.

2. We know that the Ackermann function is an example of a 1-place function which is recursive but not primitive recursive. Find an example of a 1-place predicate which is recursive but not primitive recursive.

3. Exhibit a register machine program showing that the function $f(m, n) = m^n$ is computable. (Note that $m^0 = 1$ for all $m \in \mathbb{N}$ including $m = 0$. This convention makes the recursion easier.)