

Midterm Exam  
MATH 557, Mathematical Logic

Pennsylvania State University  
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1. Use an unsigned tableau to show that

$$\neg \exists y (Sy \wedge \forall x (Exy \Leftrightarrow (Sx \wedge \neg Exx)))$$

is logically valid.

2. Let  $I$  be the identity predicate, and let  $f$  be a unary operation. Exhibit a normal structure or structures to illustrate the following facts:

- (a)  $\forall z \exists x Izfx$  is not a logical consequence of  $\forall x \forall y (Ifxfy \Rightarrow Ixy)$ .
- (b)  $\forall x Px$  is not a logical consequence of (the conjunction of)  
 $\forall x (Px \Rightarrow Pfx), \forall x \forall y (Ifxfy \Rightarrow Ixy), \exists z (Pz \wedge \neg \exists x Izfx)$ .

Try to use the simplest possible structures.

3. State König's Lemma precisely. Define all of the concepts in the lemma in terms of very basic mathematical concepts such as finite sets, infinite sets, functions, binary relations, and the natural number system.
4. Define what is meant by a quasitautology. Letting  $A$  be the sentence  $((\forall x Px) \wedge (\forall x Qx)) \Rightarrow \exists x (Px \wedge Qx)$ , exhibit a companion sequence for  $A$  which has  $A$  as a quasitautological consequence.
5. In Chapter 3 we studied several proof systems for predicate calculus:  $LH, LH', LG, LG^+, LG', LG(\text{atomic}), LG(\text{symmetric})$ . Select one of these systems. For the system that you select:
- (a) Define what are the objects of the system.
  - (b) List the axioms and rules of the system.
  - (c) Explain what is meant by a *derivation* or *proof* in the system, and what it means for an object of the system to be *derivable* or *provable*.
  - (d) Explain what it means for an object of the system to be logically valid.
  - (e) State the soundness and completeness theorems for the system.