

# Suggested research topics for Math 497A: Computability, Unsolvability, Randomness

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1. recursive real numbers, etc.
2. unsolvable mathematical problems: Hilbert's 10th Problem
3. unsolvable mathematical problems: the word problem for groups, the triviality problem for groups, etc.
4. unsolvable problems in other branches of mathematics: calculus, geometry, algebra, etc.
5. acceptable enumerations of the partial recursive functions
6. alternative methods of Gödel numbering
7. creative sets and Gödel's Incompleteness Theorem
8. simple sets
9. using partial functions as oracles
10. the structure of the Turing degrees
11. properties of the Turing jump operator
12. recursively enumerable sets and their Turing degrees
13. extensions of the arithmetical hierarchy

14. mass problems, Medvedev degrees, Muchnik degrees
15. diagonal non-recursiveness
16. basis theorems
17. other topics related to computability and unsolvability
18. weak randomness, strong randomness,  $n$ -randomness
19. genericity, weak genericity, strong genericity,  $n$ -genericity
20. tests for randomness: strong law of large numbers, law of the iterated logarithm, etc.
21. alternative notions of randomness (von Mises, Kolmogorov/Loveland, etc.)
22. randomness relative to an oracle
23.  $C$  versus  $K$ : complexity versus prefix-free complexity
24. initial segment complexity relative to an oracle
25. aspects of initial segment complexity
26. effective Hausdorff dimension
27. other topics related to randomness
28. other topics related to Kolmogorov complexity