Computability, Unsolvability, Randomness
Math 497A: Midterm Exam (6 problems)

Stephen G. Simpson

Wednesday, October 3, 2007

1. Explicitly exhibit a set which is Π^0_5 and not Σ^0_5.

2. We have seen that, given a 1-place partial recursive function ψ which is one-to-one, the inverse function ψ^{-1} is again partial recursive. The Uniformity Principle tells us that, given an index of ψ, we should expect to be able to compute an index of ψ^{-1}.

   (a) Give a rigorous statement of this result concerning indices.

   (b) Give a full proof of this result, using the Parametrization Theorem.

3. Let A and B be subsets of $\mathbb{N}$. If A and B are simple, prove that $A \cap B$ is simple.

4. Let A, B, C be recursively enumerable subsets of $\mathbb{N}$ such that $A = B \cup C$ and $B \cap C = \emptyset$. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be the respective Turing degrees of A, B, C. Prove that $\mathbf{a} = \sup(\mathbf{b}, \mathbf{c})$.

5. Consider the sets $R = \{e \mid W_e \text{ is recursive}\}$, $C = \{e \mid W_e \text{ is creative}\}$, and $S = \{e \mid W_e \text{ is simple}\}$. What can you say or guess in the way of classifying R, C and S in the arithmetical hierarchy? Prove as much as you can.

6. (a) Let f_i, $i = 0, 1, 2, \ldots$ be a countable sequence of nonrecursive total 1-place functions. Use the method of finite approximation to construct a nonrecursive total 1-place function g such that $f_i \not\leq_T g$ for all i.

   (b) Deduce that for any Turing degree $\mathbf{a} > \mathbf{0}$ we can find a Turing degree $\mathbf{b} > \mathbf{0}$ such that $\inf(\mathbf{a}, \mathbf{b}) = \mathbf{0}$.