

# Computability, Unsolvability, Randomness

## Math 497A: Homework #13

Stephen G. Simpson

Due Monday, December 3, 2007

Recall that  $A$  is said to be *strongly random* if  $A$  does not belong to any  $\Pi_2^0$  set of measure 0.

1. (a) Suppose  $A \oplus B$  is strongly random. Prove that  $\inf(\mathbf{a}, \mathbf{b}) = \mathbf{0}$  where  $\mathbf{a} = \deg_T(A)$  and  $\mathbf{b} = \deg_T(B)$ .  
(b) What if we assume only that  $A \oplus B$  is random?
2. Prove the following. If  $A$  is random and  $A \leq_T B$  and  $B$  is strongly random, then  $A$  is strongly random.
3. Prove that the following conditions are equivalent.
  - (a)  $A$  is random relative to  $0^{(n)}$  for all  $n$ .
  - (b)  $A$  does not belong to any arithmetical set of measure 0.

In this case we say that  $A$  is *arithmetically random*.