# Midterm Exam #2 – MATH 457

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This is a 50-minute exam. While taking this exam, you may not consult any books, notes, or electronic devices.

1. Write a register machine program which computes the 1-place number-theoretic function $F(n)$ = the $n$th Fibonacci number. The precise definition of $F$ is as follows:

$$F(0) = F(1) = 1,\ F(n+2) = F(n) + F(n+1).$$

2. Consider the 1-place partial number-theoretic function $\psi(n) = n/p$ where $p$ is the largest prime divisor of $n$. Prove that $\psi$ is partial computable.

   Notes:

   (a) There is no need to write a register machine program.

   (b) In your proof that $\psi$ is partial computable, you may use known lemmas and theorems, for instance our lemmas concerning bounded quantification, the least number operator, etc.

   (c) $\psi(0)$ and $\psi(1)$ are undefined.

3. Write a sentence $A$ such that the spectrum of $A$ is the set of powers of two. In other words, your sentence should have the property

$$\mathrm{spec}(A) = \{2^n \mid n = 0, 1, 2, \ldots\} = \{1, 2, 4, 8, 16, \ldots\}.$$