Midterm Exam #1 – MATH 457

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This is a 50-minute exam. While taking this exam, you may not consult any books, notes, or electronic devices.

1. Let L be the language consisting of one 2-place predicate, R. Write an L-sentence A with the following properties.

   (a) There exists at least one L-structure which satisfies A and contains exactly 5 elements.

   (b) There is no L-structure which satisfies A and contains less than 5 elements.

Which of the following sentences are logically valid?
In each case, use a structure or a tableau to justify your answer.

2. $(\forall x\, \exists y\, Rxy) \Rightarrow (\exists x\, \forall y\, Rxy)$.

3. $(\forall x\, (Hx \Rightarrow Mx)) \Leftrightarrow \neg\, \exists x\, (Hx \wedge \neg\, Mx)$.

4. $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$.

5. $((P \Rightarrow Q) \Rightarrow R) \Leftrightarrow (P \Rightarrow (Q \Rightarrow R))$.

6. $(\exists x\, (A \Rightarrow B)) \Leftrightarrow ((\forall x\, A) \Rightarrow B)$ where x does not occur in B.