Solutions to graded exercises in Homework #9

Stephen G. Simpson

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These exercises are from §§ 2.9, 3.1, 3.2 in the textbook.

§2.9 Ex. 12. The row echelon form of A shows pivot positions in columns 1, 3, and 5. Consequently, there is a basis of $\operatorname{Col} A$ consisting of columns 1, 3, 5 of A, i.e., the vectors

$$\begin{bmatrix} 1 \\ 5 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -9 \\ -9 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 7 \\ -6 \end{bmatrix},$$

so $\dim \operatorname{Col} A = 3$ and $\dim \operatorname{Nul} A = 5 - \dim \operatorname{Col} A = 2$. Additional row reduction shows that the reduced row echelon form of A is

$$\begin{bmatrix} 1 & 2 & 0 & -5 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

so there is a basis for $\operatorname{Nul} A$ consisting of the vectors

$$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}.$$

§3.1 Ex. 12. Using cofactor expansion along the first row, we have

$$\begin{vmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{vmatrix} = 4 \cdot \begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} = 4 \cdot (-1) \cdot \begin{vmatrix} 3 & 0 \\ 4 & -3 \end{vmatrix}$$

$= 4 \cdot (-1) \cdot 3 \cdot (-3) = 36$. This illustrates why the determinant of a triangular matrix is the product of the diagonal entries.

§3.1 Ex. 24. The determinants of the two given matrices are $2a-6b+3c$ and $-2a+6b-3c$ respectively. This is an example of how interchanging two rows changes the sign of the determinant.

§3.1 Ex. 40. (a) False. Cofactor expansion along any row or column gives the same result, namely, $\det A$. See Theorem 1 in §3.1.

(b) False. The determinant of a triangular matrix is the *product* of the diagonal entries. See Theorem 2 in §3.1.

§3.2 Ex. 10. THIS EXERCISE WAS NOT GRADED. Using row replacement operations only, we can show that the given determinant is equal to

$$\begin{vmatrix} 1 & 3 & -1 & 0 & -1 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & -4 & 7 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

and then a row interchange shows that this is equal to

$$-\begin{vmatrix} 1 & 3 & -1 & 0 & -1 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & -4 & 7 & -7 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = -(1)(2)(-4)(3)(1) = 24$$

since the determinant is now triangular.

§3.2 Ex. 22. By cofactor expansion along the first column, the determinant of the given matrix is

$$5 \cdot \begin{vmatrix} -3 & -2 \\ 5 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 0 & -1 \\ 5 & 3 \end{vmatrix} = 5 \cdot 1 - 1 \cdot 5 = 0.$$

Therefore, the matrix is not invertible.