Solutions to graded exercises in Homework #6
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These exercises are from §§ 2.1, 2.2, and 2.3 of the textbook.

§2.1 Ex. 18. If the first two columns of B are equal, the first two columns of AB are equal. The reason is that the ith entry of the first column of AB is simply the dot product of the ith row of A with the first column of B. And similarly for the second column of AB, and all the other columns of AB.

§2.1 Ex. 22. Let $\mathbf{b}_1, \ldots, \mathbf{b}_p$ be the columns of B. Then, the columns of AB are $A\mathbf{b}_1, \ldots, A\mathbf{b}_p$. If $\mathbf{b}_1, \ldots, \mathbf{b}_p$ are linearly dependent, let $\mathbf{x}$ be a nonzero vector in $\mathbb{R}^p$ such that $x_1\mathbf{b}_1 + \cdots + x_p\mathbf{b}_p = \mathbf{0}$. Then $x_1 A\mathbf{b}_1 + \cdots + x_p A\mathbf{b}_p = A(x_1\mathbf{b}_1 + \cdots + x_p\mathbf{b}_p) = A\mathbf{0} = \mathbf{0}$. Thus $A\mathbf{b}_1, \ldots, A\mathbf{b}_p$ are again linearly dependent.

§2.2 Ex. 10.

(a) False. The inverse of the product is the product of the inverses *in reverse order.*

(b) True. $\left(A^{-1}\right)^{-1} = A$.

(c) True. If $ad - bc = 0$ then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is not invertible.

(d) True. In fact, the row operations which reduce A to I also reduce I to A^{-1}.

(e) False. For example, let A be the 1×1 matrix $[2]$. The row operation which reduces A to $I_1 = [1]$ reduces $A^{-1} = [0.5]$ to $[0.25]$.

§2.2 Ex. 30. Let $A = \begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix}$. To find A^{-1} we perform row reduction on the matrix $[A \mid I] = \begin{bmatrix} 5 & 10 & 1 & 0 \\ 4 & 7 & 0 & 1 \end{bmatrix}$. The reduced echelon form of $[A \mid I]$ is $\left[I \mid A^{-1}\right] = \begin{bmatrix} 1 & 0 & -1.4 & 2 \\ 0 & 1 & 0.8 & -1 \end{bmatrix}$. Thus $A^{-1} = \begin{bmatrix} -1.4 & 2 \\ 0.8 & -1 \end{bmatrix}$.

§2.6 Ex. 16. If a 5×5 matrix is invertible, its columns must span $\mathbb{R}^5$. The reason is item g in Theorem 8, which says that the linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ given by the matrix is onto.