Solutions to graded exercises in Homework #5

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These exercises are from §§ 1.8 and 1.9 of the textbook.

§1.8 Ex. 11. Applying row reduction to the augmented matrix $[A, \mathbf{b}]$, we see that the pivot columns are columns 1 and 2. In particular, the rightmost column is not a pivot column. Thus the system $A\mathbf{x} = \mathbf{b}$ is consistent, i.e., $\mathbf{b}$ is in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$.

§1.8 Ex. 22.
(a) True.
(b) False. The *range* of $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A.
(c) False. The mentioned question is an *existence* question, not a uniqueness question.
(d) True. This is just the definition of a linear transformation.
(e) True.

§1.9 Ex. 18. The desired matrix is

$$\begin{bmatrix} -3 & 2 \\ 1 & -4 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

§1.9 Ex. 22. Essentially we are being asked to solve the system $x_1 - 2x_2 = -1$, $-x_1 + 3x_2 = 4$, $3x_1 - 2x_2 = 9$. The first two equations give $x_2 = 3$, and then the last equation gives $x_1 = 5$. Thus, the unique solution of $T(x_1, x_2) = (-1, 4, 9)$ is $(5, 3)$.

§1.9 Ex. 32. The correct statement is, $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto if and only if the standard matrix A has m pivot columns. The reason is that A is an $m \times n$ matrix, so having m pivot columns is the same as having a pivot position in every row. By Theorem 4 in §1.4 this is equivalent to the columns of A spanning $\mathbb{R}^m$, which by Theorem 12 in §1.9 is equivalent to T being onto $\mathbb{R}^m$.