Solutions to graded exercises in Homework #4
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February 7, 2011

These exercises are from §§ 1.5, 1.7, and 1.8 of the textbook.

§1.5 Ex. 30. Let A be a 3×3 matrix with two pivot positions.

(a) Yes, $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution. This is because the system has a free variable, because A has a non-pivot column. (See the highlighted text on page 50.)

(b) No, $A\mathbf{x} = \mathbf{b}$ does not have a solution for all $\mathbf{b}$. This is because A has a non-pivot row. (See Theorem 4 on page 43.)

§1.7 Ex. 6. Let A be the given 4×3 matrix. Row reduction shows that all of the columns of A are pivot columns. Hence $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. (See the highlighted text on page 50.) In other words, the columns of A are linearly independent.

§1.7 Ex. 21. (a) False. A homogenous system *always* has the trivial solution.

(b) False. For instance, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are linearly dependent, but the second vector is not a linear combination of the first.

(c) True. Any 5 vectors in $\mathbb{R}^4$ are linearly dependent.

(d) True. If $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are linearly dependent, consider a dependence relation among them. The coefficient of $\mathbf{z}$ must be nonzero, because otherwise $\mathbf{x}$ and $\mathbf{y}$ would be linearly dependent. We can then solve for $\mathbf{z}$ to express $\mathbf{z}$ as a linear combination of $\mathbf{x}$ and $\mathbf{y}$. In other words, $\mathbf{z}$ belongs to the span of $\mathbf{x}$ and $\mathbf{y}$.

§1.7 Ex. 32. We have $1\cdot$(column 1) + $2\cdot$(column 2) + $(-1)\cdot$(column 3) = $\mathbf{0}$. In other words, the vector

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$.

§1.8 Ex. 4. The problem is to find a vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$. Applying row reduction to the augmented matrix $[A, \mathbf{b}]$, we get the reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

which gives

$$\mathbf{x} = \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$$

as a solution of the nonhomogeneous system $A\mathbf{x} = \mathbf{b}$. Since every row of A is a pivot row, the homogeneous system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. From this it follows that our solution of the nonhomogeneous system is unique.