Solutions to graded exercises in Homework #3

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These exercises are from §§ 1.3 and 1.4 of the textbook.

§1.3 Ex. 24.
- (a) True.
- (b) True. $\mathbf{v} + (\mathbf{u} - \mathbf{v}) = \mathbf{u}$.
- (c) False. The weights in a linear combination can be any scalars, including 0.
- (d) True.
- (e) True.

§1.3 Ex. 26.
- (a) The augmented matrix $[\,A, \mathbf{b}\,]$ is

$$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix}$$

which easily reduces to a (non-unique) row echelon form, for instance

$$\begin{bmatrix} 2 & 0 & 6 & 10 \\ 0 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In this way we see that the pivot columns are columns 1 and 2. Since the rightmost column is not a pivot column, it follows that the system $A\mathbf{x} = \mathbf{b}$ has at least one solution. In other words, $\mathbf{b}$ is a linear combination of the columns of A.

- (b) Trivially the third column (or any column) of A is a linear combination of the columns of A. (This holds for any matrix.)

§1.3 Ex. 29. The center of mass is $(1.3, .9, 0)$. The student must show the work to obtain this vector as a weighted average of the given vectors.

§1.4 Ex. 10. The vector equation is

$$x_1 \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

and the matrix equation is

$$\begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}.$$

§1.4 Ex. 17. The student should put A into row echelon form in order to find the pivot positions. After doing this, we see that the pivot positions are: row 1 column 1, row 2 column 2, and row 3 column 4. Since row 4 does not contain a pivot position, it follows by Theorem 4 in §1.4 that the system $A\mathbf{x} = \mathbf{b}$ is not consistent for all $\mathbf{b}$.