Solutions to graded exercises in Homework #2

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These exercises are from Sections 1.2 and 1.3 of the textbook.

§1.2 Ex. 3. The given matrix is row-equivalent to

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which is an RREF or reduced echelon matrix. (The student must show how this matrix was obtained.) In both matrices, the pivot positions are: row 1 column 1, and row 2 column 2. In both matrices, the pivot columns are columns 1 and 2.

§1.2 Ex. 15. System (a) is consistent, because the last column is not a pivot column. The solution of system (a) is unique, because there are no free variables. System (b) is inconsistent, because the last column is a pivot column.

§1.2 Ex. 22.
(a) False. The reduced echelon form (RREF) of a given matrix is unique, but the echelon form (REF) is not.
(b) False. All row operations leave the pivot positions unchanged.
(c) True.
(d) True.
(e) True.

§1.2 Ex. 29. In any linear system, the number of basic variables is at most the number of equations. Hence, in a system with more equations than variables, there must be at least one non-basic variable, a.k.a., free variable. Thus, if such a system is consistent, there are infinitely many solutions, corresponding to infinitely many values of the free variables.

§1.3 Ex. 14. The augmented matrix $[A, \mathbf{b}]$ is

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix}.$$

A row operation transforms this to an echelon or REF form

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{bmatrix}.$$

Thus we see that the pivot columns are columns 1, 2, and 3. Since the rightmost column is not a pivot column, the associated linear system is consistent. In other words, **b** is indeed a linear combination of the columns of A.