Solutions to graded exercises in Homework #10
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These exercises are from §5.1 in the textbook.

§5.1 Ex. 4. We have

$$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} -1+2\sqrt{2} \\ 3+\sqrt{2} \end{bmatrix} = (3+\sqrt{2}) \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$$

which means that $\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$ and the corresponding eigenvalue is $3+\sqrt{2}$.

§5.1 Ex. 8. Let A be the given matrix. By row reduction we have

$$A - 3I = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -5 & 1 \\ 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

so the general solution of $A\mathbf{x} = 3\mathbf{x}$ is

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

Thus $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to the eigenvalue 3.

§5.1 Ex. 14. By row reduction we have

$$A + 2I = \begin{bmatrix} 3 & 0 & -1 \\ 1 & -1 & 0 \\ 4 & -13 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & 0 \end{bmatrix}$$

so the general solution of $A\mathbf{x} = -2\mathbf{x}$ is

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1/3 \\ 1/3 \\ 1 \end{bmatrix}.$$

Thus either of the vectors $\begin{bmatrix} 1/3 \\ 1/3 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ is by itself a basis of the eigenspace of A corresponding to the eigenvalue -2.

§5.1 Ex. 16. By row reduction we have

$$A - 4I = \begin{bmatrix} -1 & 0 & 2 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so the general solution of $A\mathbf{x} = 4\mathbf{x}$ is

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Thus the vectors $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ are a basis of the eigenspace of A corresponding to the eigenvalue 4.

§5.1 Ex. 32. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a rotation about a line through the origin. Then 1 is an eigenvalue of T, and the corresponding eigenspace consists of all vectors in $\mathbb{R}^3$ which are parallel to the given line.