Solutions to graded exercises in Homework #1
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These exercises are from Section 1.1 of the textbook.

5. Replace (row 1) by (row 1) - 5·(row 3).
   Replace (row 2) by (row 2) +3·(row 3).

9. The given matrix is in *row echelon form* and tells us that there is a unique solution. Using elementary row operations, the student should bring this into *reduced row echelon form*

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

   which tells us that the unique solution is $x_1 = 4$, $x_2 = 8$, $x_3 = 5$, $x_4 = 2$.

18. The augmented matrix for the equations of the three planes is

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & -3 \\ -1 & -3 & 0 & 4 \end{bmatrix}.$$

   Using elementary row operations, the student should transform this to

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

   which is the augmented matrix of the system $x_1 + 2x_2 + x_3 = 4$, $x_2 - x_3 = -3$, $0 = 5$. The equation $0 = 5$ tells us that this system is inconsistent, which means that the three planes have no point in common.

24. (a) True.

    (b) False. Row equivalence means the matrices can be transformed into each other by elementary row operations.

(c) False. An inconsistent system has *no* solution.

(d) True.

34. The equations are
$$T_1 = \frac{10 + 20 + T_2 + T_4}{4},$$
$$T_2 = \frac{T_1 + 20 + 40 + T_3}{4},$$
$$T_3 = \frac{T_4 + T_2 + 40 + 30}{4},$$
$$T_4 = \frac{10 + T_1 + T_3 + 30}{4}.$$

The unique solution of this system is $T_1 = 20$, $T_2 = 27.5$, $T_3 = 30$, $T_4 = 22.5$. In reaching this solution, the students are required to show their work.