MATH 220	NAME ______________________
EXAM I	STUDENT NUMBER ______________________
SPRING 2011	INSTRUCTOR ______________________
VERSION A	SECTION NUMBER ______________________

On your scantron, write and bubble your PSU ID, Section Number, and Test Version. Failure to correctly code these items may result in a loss of 5 points on the exam.

On your scantron, bubble letters corresponding to your answers on indicated questions. It is a good idea for future review to circle your answers in the test booklet.

Check that your exam contains 20 multiple-choice questions, numbered sequentially.

Answer Questions 1–20 on your scantron.

Each question is worth 5 points.

THE USE OF A CALCULATOR, CELL PHONE, OR ANY OTHER ELECTRONIC DEVICE IS NOT PERMITTED IN THIS EXAMINATION.

THE USE OF NOTES OF ANY KIND IS NOT PERMITTED DURING THIS EXAMINATION.

1. Solve the linear system

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= -5 \\ 3x_1 + 2x_2 - 6x_3 &= 3 \\ -x_1 + x_2 &= 6 \end{aligned}$$

a) $\left(-\frac{53}{3}, -\frac{35}{3}, -\frac{56}{9}\right)$

b) $\left(1, 5, \frac{2}{3}\right)$

c) $\left(-11, -5, \frac{2}{3}\right)$

⋆d) $\left(-1, 5, \frac{2}{3}\right)$

2. How many solutions does the following system have?

$$\begin{aligned} 3x_1 + 5x_2 + 4x_3 &= -1 \\ 2x_1 - 2x_2 + x_3 &= 0 \\ 7x_1 + x_2 + 6x_3 &= 2 \end{aligned}$$

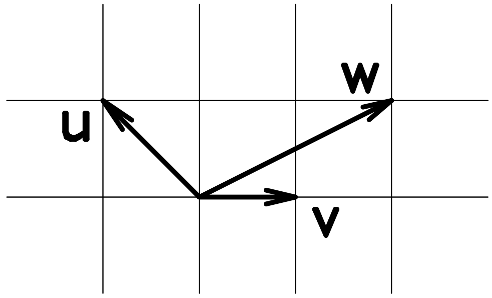
⋆a) None

b) One

c) Infinitely many, with 1 free variable

d) Infinitely many, with 2 free variables

3. According to the graph



how can we best describe the vector $\mathbf{w}$ in terms of the vectors $\mathbf{u}$ and $\mathbf{v}$?

a) $2\mathbf{v} + \mathbf{u}$

b) $3\mathbf{v} - \mathbf{u}$

⋆c) $3\mathbf{v} + \mathbf{u}$

d) $2\mathbf{v} - \mathbf{u}$

4. For which value(s) of h is $\mathbf{b} = \begin{bmatrix} 0 \\ h \end{bmatrix}$ in the set spanned by the vectors $\mathbf{a}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$?

a) $h = 0$

b) $h = 1$

c) $h = 2$

⋆d) $h =$ any real number.

5. Which of the following is a linearly independent set?

⋆a) $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 4 \\ 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1.5 \\ 0 \end{bmatrix}$

c) $\mathbf{v}$, $2\mathbf{v} + \mathbf{u}$, $\mathbf{0}$

d) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

6. What is the standard matrix for rotation of the xy-plane about the origin by 180°?

a) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

⋆c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

7. Which of the following matrices is invertible?

a) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

⋆c) $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 10 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

8. Find a basis of the column space Col A for the matrix

$$A = \begin{bmatrix} 2 & 3 & -2 & 1 & 4 \\ 4 & 5 & -3 & 11 & 4 \\ 4 & 6 & -4 & 3 & 4 \end{bmatrix}.$$

a) $\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}$

⋆b) $\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 11 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 11 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$

d) $\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$

9. The following are augmented matrices of corresponding linear systems. Which system is consistent when $\mathbf{b} = [b_1, b_2, b_3]^T$ is any vector in $\mathbb{R}^3$?

a) $\begin{bmatrix} 1 & 2 & 3 & b_1 \\ 4 & 5 & 6 & b_2 \\ 7 & 8 & 9 & b_3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 & b_1 \\ 4 & 5 & 6 & b_2 \\ 6 & 8 & 10 & b_3 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & -4 & b_1 \\ 2 & -3 & 2 & b_2 \\ 4 & -5 & 0 & b_3 \end{bmatrix}$

⋆d) $\begin{bmatrix} 1 & 3 & 4 & b_1 \\ -4 & 2 & 6 & b_2 \\ -3 & -2 & -7 & b_3 \end{bmatrix}$

10. Which of the following multiplications will give you a vector in $\mathbb{R}^4$?

a) $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

b) $\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$

$\star$c) $\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 8 \\ 9 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & -4 & 3 \\ 2 & -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$

11. Which of the following is *not* a linear transformation?

a) $T(x) = 3x$

b) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

c) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 + x_2$

$\star$d) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = xy$

12. Find the entries in the second row of AB where $A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$.

⋆a) $[5, 1]$

b) $[5, -1]$

c) $[2, 3]$

d) $[2, -3]$

13. Which of following matrices is in reduced echelon form?

a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

⋆d) $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

14. Find the parametric vector form of the solution set of the matrix equation $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 1 & -1 & 0 & 3 & 3 \\ 0 & 0 & 1 & -5 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a) $\mathbf{x} = x_1 \begin{bmatrix} -1 \\ 0 \\ 4 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 2 \\ 5 \end{bmatrix}$

⋆b) $\mathbf{x} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 0 \\ 5 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$

c) $\mathbf{x} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$

d) $\mathbf{x} = x_2 \begin{bmatrix} -3 \\ 0 \\ 5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}$

15. Consider the matrix $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ -3 & -9 & -15 \end{bmatrix}$. Which of the following vectors belong to the null space of A?

a) All vectors in $\mathbb{R}^3$.

⋆b) All vectors $\mathbf{v}$ in $\mathbb{R}^3$ such that $v_1 = v_3 = -\frac{v_2}{2}$.

c) $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

d) The zero vector only.

16. Suppose A is an $m \times n$ matrix and all of its columns are pivot columns. Then, which of the following statements is *not necessarily true*?

a) The equation $A\mathbf{x} = \mathbf{b}$ has either a unique solution or none at all, for each vector $\mathbf{b}$ in $\mathbb{R}^m$.

⋆b) The equation $A\mathbf{x} = \mathbf{b}$ is consistent for any vector $\mathbf{b}$ in $\mathbb{R}^m$.

c) The set of column vectors of A is a linearly independent set.

d) The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

17. Let T be a linear transformation such that $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}2\\3\end{bmatrix}$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\end{bmatrix}$. Find $T\left(\begin{bmatrix}4\\5\end{bmatrix}\right)$.

a) $\begin{bmatrix}3\\9\end{bmatrix}$

b) $\begin{bmatrix}11\\9\end{bmatrix}$

c) $\begin{bmatrix}-6\\5\end{bmatrix}$

⋆d) $\begin{bmatrix}9\\11\end{bmatrix}$

18. If $(A^{-1})^T = \begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}$, what is A?

a) $\begin{bmatrix}2 & -0.3\\-0.2 & 0.1\end{bmatrix}$

b) $\begin{bmatrix}2 & -3\\1 & 1\end{bmatrix}$

⋆c) $\begin{bmatrix}0.4 & -0.3\\0.2 & 0.1\end{bmatrix}$

d) $\begin{bmatrix}4 & -3\\2 & 1\end{bmatrix}$

19. Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make $AB = BA$?

a) 4

⋆b) 5

c) 6

d) Any real number

20. Suppose T is the linear transformation given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -5x_1 + 9x_2 \\ 4x_1 - 7x_2 \end{bmatrix}$. Is T one-to-one? Is T onto? Can you find T^{-1}?

a) T is one-to-one but not onto.

b) T is onto but not one-to-one.

c) T is one-to-one and onto, and $T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -7x_1 - 9x_2 \\ -4x_1 - 5x_2 \end{bmatrix}$.

⋆d) T is one-to-one and onto, and $T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 7x_1 + 9x_2 \\ 4x_1 + 5x_2 \end{bmatrix}$.