MATH 220 NAME ____________________

EXAM 2 STUDENT NUMBER ____________________

SPRING 2011 INSTRUCTOR ____________________

VERSION A SECTION NUMBER ____________________

On your scantron, write and bubble your PSU ID, Section Number, and Test Version. Failure to correctly code these items may result in a loss of 5 points on the exam.

On your scantron, bubble letters corresponding to your answers on indicated questions. It is a good idea for future review to circle your answers in the test booklet.

Check that your exam contains 30 multiple-choice questions, numbered sequentially.

Answer Questions 1–30 on your scantron.

Each question is worth 5 points.

THE USE OF A CALCULATOR, CELL PHONE, OR ANY OTHER ELECTRONIC DEVICE IS NOT PERMITTED IN THIS EXAMINATION.

THE USE OF NOTES OF ANY KIND IS NOT PERMITTED DURING THIS EXAMINATION.

1. Determine h and k such that the solution set of the following system is empty.

$$x_1 + 3x_2 = k$$
$$4x_1 + hx_2 = 8$$

⋆a) $h = 12$, $k \neq 2$

b) $h = 12$, $k = 2$

c) $h \neq 12$, $k \neq 2$

d) $h \neq 12$, $k = 2$

2. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$. For what value(s) of h is $\mathbf{y}$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$?

a) $h = \frac{3}{2}$

b) $h = \frac{1}{4}$

⋆c) $h = -\frac{7}{2}$

d) All real numbers

3. Suppose $AB = \begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$. Find A.

a) $A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$

b) $A = \begin{bmatrix} 11 & -17 \\ -27 & 41 \end{bmatrix}$

c) $A = \begin{bmatrix} 13 & 43 \\ 4 & 18 \end{bmatrix}$

⋆d) $A = \begin{bmatrix} -3 & 13 \\ -8 & 27 \end{bmatrix}$

4. If $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 8$, what is $\det\begin{bmatrix} a & b & c \\ 3g+a & 3h+b & 3i+c \\ d & e & f \end{bmatrix}$?

a) 8

$\star$b) -24

c) 24

d) -8

5. Find the determinant of

$$\begin{bmatrix} 9 & 1 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 2 \\ 4 & 0 & 0 & 5 & 0 \\ 9 & 0 & 3 & 9 & 0 \\ 6 & 0 & 0 & 7 & 0 \end{bmatrix}.$$

a) 0

b) -2

$\star$c) -12

d) 12

6. Let A be an invertible $n \times n$ matrix. Which of the following statements is not necessarily true?

a) The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.

b) A^T is invertible.

c) $\text{Col } A = \mathbb{R}^n$.

$\star$d) The number 0 is an eigenvalue of A.

7. Let $A = \begin{bmatrix} 6 & 1 \\ 3 & 4 \end{bmatrix}$. Which of the following matrices is similar to A?

⋆a) $\begin{bmatrix} 7 & 0 \\ 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

d) None of the above.

8. Which of the following is a unit vector in the same direction as $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$?

a) $\begin{bmatrix} -\sqrt{6} \\ \sqrt{6}/3 \\ \sqrt{6} \end{bmatrix}$

⋆b) $\begin{bmatrix} -\sqrt{6}/6 \\ \sqrt{6}/3 \\ \sqrt{6}/6 \end{bmatrix}$

c) $\begin{bmatrix} -1/6 \\ 1/3 \\ 1/6 \end{bmatrix}$

d) $\mathbf{v}$ is already a unit vector.

9. Which of the following statements is always true?

a) An $n \times n$ symmetric matrix has n distinct real eigenvalues.

b) An orthogonal matrix is orthogonally diagonalizable.

c) If P is an $n \times n$ matrix with orthogonal columns, then $P^T = P^{-1}$.

⋆d) Every symmetric matrix is orthogonally diagonalizable.

10. Find the matrix of the quadratic form $8x_1^2 + 7x_2^2 - 3x_3^2 - 6x_1x_2 + 4x_1x_3 - 2x_2x_3$.

⋆a) $\begin{bmatrix} 8 & -3 & 2 \\ -3 & 7 & -1 \\ 2 & -1 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 8 & -6 & 4 \\ 0 & 7 & -2 \\ 0 & 0 & -3 \end{bmatrix}$

c) $\begin{bmatrix} -3 & 4 & 7/2 \\ 4 & 2 & -3/2 \\ 7/2 & -3/2 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 8 & 2 & -3 \\ 2 & 7 & -1 \\ -3 & -1 & -3 \end{bmatrix}$

11. Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the data points $(0, 1), (1, 1), (2, 2), (3, 2)$.

⋆a) $y = .9 + .4x$

b) $y = 4.5 + 9.5x$

c) $y = .4 + .9x$

d) $y = 9.5 - 4.5x$

12. Find the characteristic polynomial of the following matrix:

$$A = \begin{bmatrix} -2 & 0 & 3 \\ 1 & 5 & -1 \\ 2 & 0 & 4 \end{bmatrix}.$$

a) $(-2-\lambda)(5-\lambda)(4-\lambda)$

⋆b) $(5-\lambda)(\lambda^2-2\lambda-14)$

c) $\lambda^3+5\lambda^2+6\lambda-14$

d) $5(\lambda^2-2\lambda-14)$

13. For what value of h are the vectors $\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} h \\ 5 \\ 6 \end{bmatrix}$ linearly independent?

a) $h \neq 12$

⋆b) $h \neq -12$

c) $h = -12$

d) $h = -10$

14. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$. What is $T^{-1}\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$?

a) $\begin{bmatrix} 8 \\ 10 \end{bmatrix}$

b) $\begin{bmatrix} -8 \\ 10 \end{bmatrix}$

⋆c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

d) $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$

15. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, find the second column of A^{-1}.

a) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

⋆b) $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

d) $\frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

16. Find the coordinate vector of $\mathbf{x}$ relative to the basis $\{\mathbf{v_1}, \mathbf{v_2}\}$ where $\mathbf{x} = \begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix}$, $\mathbf{v_1} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} -5 \\ -2 \\ 4 \end{bmatrix}$.

⋆a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$

d) $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$

17. What is the rank of a 3×6 matrix whose null space is 4-dimensional?

a) 1

$\star$b) 2

c) 3

d) 4

18. Find the eigenvalues of $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$.

a) 1, 1, 2

b) 1, 2, 2

c) 1, -1, 2

$\star$d) 1, 2, 3

19. Given the diagonalized matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$, compute A^{10}.

a) $\frac{1}{2} \begin{bmatrix} 3^{10} - 1 & 3^{10} + 1 \\ 3^{10} + 1 & 3^{10} - 1 \end{bmatrix}$

b) $\frac{1}{2} \begin{bmatrix} 3^{10} - 1 & 3^{10} - 1 \\ 3^{10} - 1 & 3^{10} + 1 \end{bmatrix}$

$\star$c) $\frac{1}{2} \begin{bmatrix} 3^{10} + 1 & 3^{10} - 1 \\ 3^{10} - 1 & 3^{10} + 1 \end{bmatrix}$

d) $\frac{1}{2} \begin{bmatrix} 3^{10} + 1 & 3^{10} + 1 \\ 3^{10} + 1 & 3^{10} - 1 \end{bmatrix}$

20. What is the distance between $\mathbf{u} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$?

a) 3

b) $\sqrt{10}$

c) $\sqrt{12}$

⋆d) $\sqrt{11}$

21. Find the orthogonal projection of $\mathbf{x}$ onto Span$\{\mathbf{v}_1, \mathbf{v}_2\}$ where $\mathbf{x} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

⋆a) $\begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$

d) $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

22. If A is an $n \times n$ square matrix, and I denotes the $n \times n$ identity matrix, which of the following statements is not necessarily true?

$\star$a) If the columns of A form an orthogonal set, then A is an orthogonal matrix.

b) If $AA^T = I$, then A is an orthogonal matrix.

c) If A is an orthogonal matrix, its determinant must be ± 1.

d) If A is an orthogonal matrix, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ preserves lengths and orthogonality.

23. Which of the following statements is *not* true?

a) An $n \times n$ matrix that is orthogonally diagonalizable must be symmetric.

b) For any matrix A, $A^T A$ is symmetric.

c) If A is symmetric, any two eigenvectors of A corresponding to different eigenvalues must be orthogonal.

$\star$d) If A and B are symmetric matrices, AB must also be symmetric.

24. Find the least-squares solution for the system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 3 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$.

$\star$a) $\begin{bmatrix} 1/2 \\ 5/9 \end{bmatrix}$

b) $\begin{bmatrix} 1/2 \\ 10/9 \end{bmatrix}$

c) $\begin{bmatrix} 1/2 \\ 7/9 \end{bmatrix}$

d) $\begin{bmatrix} 1/2 \\ 1/9 \end{bmatrix}$

25. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Which of the following correctly express $\mathbf{v}$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$?

a) $\mathbf{v} = \dfrac{x+y+z}{2}\mathbf{v}_1 + \dfrac{x-y}{3}\mathbf{v}_2 + \dfrac{x+y-z}{5}\mathbf{v}_3$

b) $\mathbf{v} = \dfrac{x+y+z}{7}\mathbf{v}_1 + \dfrac{x+y-2z}{4}\mathbf{v}_2 + \dfrac{x-y}{2}\mathbf{v}_3$

⋆c) $\mathbf{v} = \dfrac{x+y+z}{3}\mathbf{v}_1 + \dfrac{x-y}{2}\mathbf{v}_2 + \dfrac{x+y-2z}{6}\mathbf{v}_3$

d) $\mathbf{v} = \dfrac{x-y-z}{3}\mathbf{v}_1 + \dfrac{x-y}{7}\mathbf{v}_2 + \dfrac{x-y+2z}{6}\mathbf{v}_3$

26. Let $W = \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 6 \\ 0 \end{bmatrix}$. Which of the following vectors belong to the orthogonal complement of W?

⋆a) $\begin{bmatrix} 0 \\ 8 \\ 0 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 0 \\ 17 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 12 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 17 \\ 0 \\ -1 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ 4 \\ 11 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 2 \\ -8 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 0 \\ 12 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 12 \\ 0 \\ -6 \\ 0 \end{bmatrix}$

27. Find a basis of the column space Col$A$ for the matrix $A = \begin{bmatrix} 4 & 5 & -11 & 2 & 5 \\ 4 & 5 & -3 & 11 & 7 \\ 4 & 6 & -4 & 3 & 1 \end{bmatrix}$.

a) The first and second columns

b) The first and last columns

⋆ c) The first, second and third columns

d) The fourth and fifth columns

28. Let $\mathbf{v} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$. Which of the following vectors is in the same direction as $\mathbf{v}$?

⋆ a) $\begin{bmatrix} 51 \\ -51 \\ 17 \end{bmatrix}$

b) $\begin{bmatrix} -7 \\ -7 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} -63 \\ 63 \\ -21 \end{bmatrix}$

d) $\begin{bmatrix} 57 \\ 57 \\ -17 \end{bmatrix}$

29. Let $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. What is the distance from $\mathbf{y}$ to the line through $\mathbf{0}$ and $\mathbf{v}$?

a) 1

b) $\sqrt{2}$

⋆c) $\sqrt{3}$

d) 2

30. The vectors $\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}$ form a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.

a) $\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

⋆b) $\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}$

d) $\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}$