Math 141H.1, Honors Calculus II

Midterm Exam 4

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The exam consists of eight problems. Calculators are not allowed.

1. Find a power series representation of $\sqrt{1 - 9x^4}$ centered at $x = 0$. What is the radius of convergence?

2. Find a power series representation for $\sin x$ centered at $x = \dfrac{\pi}{4}$.

3. For which x-values is the power series

$$\sum_{n=0}^{\infty} \frac{x^n}{(n+1)(n+2)(n+3)}$$

   absolutely convergent? Conditionally convergent? Divergent? Justify your answers.

4. Let $f(x)$ be the power series of the previous problem. Find $f^{(7)}(0)$, the seventh derivative of $f(x)$ evaluated at $x = 0$.

5. Find the first 4 terms of a power series representation for $e^x/(1 - x)$ centered at $x = 0$.

6. Let C be a parametrized curve defined by $x = t + \ln t$, $y = \tan^{-1} t$. Let P be the point
$$\left(\sqrt{3} + \frac{\ln 3}{2}, \ \frac{\pi}{3} \right)$$
   on C. Let L be the tangent line to C at P. Find an equation for L.

7. Let C be the curve $y = 2\sqrt{x}$, $0 \le x \le 1$. Find the area of the surface obtained by revolving C about the x-axis.

8. Use the power series method to solve the initial value problem
$$y' = 2y + 1, \qquad y(0) = 0.$$