Math 141H.1, Honors Calculus II

Midterm Exam 3

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The exam consists of ten problems. Calculators are not allowed.

In problems 1–8, determine whether the given series is absolutely convergent, conditionally convergent, or divergent. When possible, find the sum of the series. Justify your answers fully, using appropriate convergence tests.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6(3)^n} = \frac{1}{18} + \frac{1}{54} + \frac{1}{162} + \cdots$

2. $\displaystyle\sum_{n=1}^{\infty} 2^{-n/3} = 2^{-1/3} + 2^{-2/3} + 2^{-3/3} + 2^{-4/3} + \cdots$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n+4}{n\sqrt{n}} = \frac{5}{1\sqrt{1}} + \frac{6}{2\sqrt{2}} + \frac{7}{3\sqrt{3}} + \frac{8}{4\sqrt{4}} + \cdots$

4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\log_{10} n)^3} = \frac{1}{2(\log_{10} 2)^3} + \frac{1}{3(\log_{10} 3)^3} + \frac{1}{4(\log_{10} 4)^3} + \cdots$

5. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \log_{10} n} = \frac{1}{2\log_{10} 2} - \frac{1}{3\log_{10} 3} + \frac{1}{4\log_{10} 4} - \frac{1}{5\log_{10} 5} + \cdots$

6. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}} = \frac{1}{\sqrt[2]{2}} - \frac{1}{\sqrt[3]{3}} + \frac{1}{\sqrt[4]{4}} - \frac{1}{\sqrt[5]{5}} + \cdots$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{100^n}{n!} = \frac{100^1}{1!} + \frac{100^2}{2!} + \frac{100^3}{3!} + \frac{100^4}{4!} + \cdots$

8. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \left(\frac{2n+1}{3n+1}\right)^n = 1 - \left(\frac{3}{4}\right)^1 + \left(\frac{5}{7}\right)^2 - \left(\frac{7}{10}\right)^3 + \cdots$

9. Use the first five terms of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} = \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots$$

to give upper and lower bounds for the sum of the series. Express the sum of the series as a decimal, accurate to two decimal places.

10. Express the sum of the series

$$\sum_{n=1}^{\infty} \frac{3+2(-1)^n}{10^n} = \frac{1}{10} + \frac{5}{100} + \frac{1}{1000} + \frac{5}{10000} + \cdots = 0.1515\ldots$$

as a quotient of two integers.