Math 141H.1, Honors Calculus II

Midterm Exam 2

Stephen G. Simpson

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The exam consists of eight problems. Calculators are not allowed.

1. Is the improper integral

$$\int_1^\infty \frac{5 + 2\tan^{-1} x}{x^2}\, dx$$

convergent or divergent? Justify your answer.

2. Let y be the amount of money in your bank account. Assume that y is \$1000 on January 1, 2002 and grows at the rate of 6 percent per year compounded continuously. What is y on January 1, 2052? Is it more than \$10,000? Why or why not?

3. Evaluate the improper integral

$$\int_0^\infty \frac{dx}{(x+1)(x+2)} \; .$$

4. Find the indefinite integral

$$\int \frac{x^3}{x^2+1}\, dx \; .$$

5. Find the indefinite integral

$$\int (x-1)^2 \cos 2x \, dx \; .$$

6. Evaluate the limit

$$\lim_{x \to 0} \frac{x - \sin x}{x^3} \; .$$

7. Evaluate the definite integral

$$\int_0^1 \frac{x\, dx}{(x^2+1)^5} \; .$$

8. Simpson's Rule may be written as

$$\int_a^b f(x)\,dx \;\approx\; \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)\,.$$

What values of h and n are needed, in order to approximate

$$\int_2^{20} \cos x \; dx$$

to within 10^{-5} ?