Math 141H.1, Honors Calculus II

Bonus Problems 1

Stephen G. Simpson

February 1, 2002

Please work alone. You may use a calculator or computer algebra system if you wish, but please give exact solutions and show all the steps needed to obtain your solutions by hand.

1. If $f(x) = (10x)^{\log_{10} x}$, find the derivative $f'(x)$.

2. Find the centroid of the region in the xy-plane described by

$$0 \leq x \leq \frac{\pi}{2}, \qquad 0 \leq y \leq \sin x .$$

3. Let R be the region in the xy-plane described by

$$0 \leq x \leq 1, \qquad 0 \leq y \leq \frac{1}{x^4 + 2x^2 + 1} .$$

   Find the volume of the solid of revolution obtained by revolving R about the x-axis.

4. Evaluate the improper integral

$$\int_0^\infty \frac{dx}{(x + 1)(x^3 + 1)} .$$

5. Find

$$\lim_{n \to \infty} \frac{\ln(1 + 1/n) + \ln(1 + 2/n) + \cdots + \ln(1 + n/n)}{n}$$

   and justify your answer.