MATH 140H, Honors Calculus I – Fall 2011

TERM PAPER ASSIGNMENT

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Assignment

The assignment is as follows.

- Each student in this course is required to submit a term paper.
- The topic of the term paper is to be selected by the student.
- The topic should be on some aspect of the foundations of calculus.
- The term paper should be neatly typewritten and at least 5 to 10 pages long.
- Each term paper will be graded as satisfactory or unsatisfactory.
- A satisfactory term paper will result in a higher grade in the course.

Deadlines

The deadlines are as follows.

- A one-page term paper proposal or outline must be submitted electronically by November 18.
- The term paper itself must be submitted electronically by December 9.

Suggested background reading

Some references on the foundations of calculus are as follows.

- Richard Dedekind, *Essays on the Theory of Numbers*, Dover, 1963, III + 115 pages.
- Solomon Feferman, *The Number Systems: Foundations of Algebra and Analysis*, Chelsea, 1989, XII + 418 pages.

- Elliott Mendelson, *Number Systems and the Foundations of Analysis*, Academic Press, 1973, XII + 358 pages.
- Kenneth A. Ross, *Elementary Analysis: The Theory of Calculus*, Springer, 1980, X + 351 pages.

Suggested topics

Some suggested term paper topics are as follows.

- Exposition and commentary on Sections I–VI of Dedekind's essay "The Nature and Meaning of Numbers."
- Exposition and commentary on Sections VII–X of Dedekind's essay "The Nature and Meaning of Numbers."
- Exposition and commentary on Sections XI–XIV of Dedekind's essay "The Nature and Meaning of Numbers."
- Exposition and commentary on Dedekind's essay "Continuity and Irrational Numbers."
- Peano systems: the Iteration Theorem and its proof.
- Peano systems: the Isomorphism Theorem and its proof.
- Peano systems: definition and basic properties of addition, order, multiplication, and exponentiation.
- Peano systems: proofs of the basic properties of addition, order, multiplication, and exponentiation.
- The rational number system: constructing it from the natural number system; definition of addition, order, and multiplication.
- The rational number system: proofs of the basic properties of addition, order, and multiplication.
- The real number system: Dedekind cuts.
- The real number system: Cauchy sequences.
- The real number system: proofs of the basic properties of addition, order, and multiplication.
- Proof of the Heine-Borel Theorem.
- Continuous functions: rigorous definition and basic properties.
- Continuous functions: proof of the Extreme Value Theorem.
- Continuous functions: proof of the existence of the Riemann integral.
- Continuous functions: proof of the Fundamental Theorem of Calculus.