

36. $f''(t) = 3/\sqrt{t} = 3t^{-1/2} \Rightarrow f'(t) = 6t^{1/2} + C$. $f'(4) = 12 + C$ and $f'(4) = 7 \Rightarrow C = -5$, so $f'(t) = 6t^{1/2} - 5$ and hence, $f(t) = 4t^{3/2} - 5t + D$. $f(4) = 32 - 20 + D$ and $f(4) = 20 \Rightarrow D = 8$, so $f(t) = 4t^{3/2} - 5t + 8$.

42. $f'(x) = x^3 \Rightarrow f(x) = \frac{1}{4}x^4 + C$. $x + y = 0 \Rightarrow y = -x \Rightarrow m = -1$. Now $m = f'(x) \Rightarrow -1 = x^3 \Rightarrow x = -1 \Rightarrow y = 1$ (from the equation of the tangent line), so $(-1, 1)$ is a point on the graph of f . From f , $1 = \frac{1}{4}(-1)^4 + C \Rightarrow C = \frac{3}{4}$. Therefore, the function is $f(x) = \frac{1}{4}x^4 + \frac{3}{4}$.

66. $v'(t) = a(t) = -22$. The initial velocity is 50 mi/h = $\frac{50 \cdot 5280}{3600} = \frac{220}{3}$ ft/s, so $v(t) = -22t + \frac{220}{3}$.

The car stops when $v(t) = 0 \Leftrightarrow t = \frac{220}{3 \cdot 22} = \frac{10}{3}$. Since $s(t) = -11t^2 + \frac{220}{3}t$, the distance covered is

$$s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3} \cdot \frac{10}{3} = \frac{1100}{9} = 122.\bar{2} \text{ ft.}$$

22. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \left(5 + \frac{2i}{n}\right)^{10}$ can be interpreted as the area of the region lying under the graph of $y = (5 + x)^{10}$ on the interval

$[0, 2]$, since for $y = (5 + x)^{10}$ on $[0, 2]$ with $\Delta x = \frac{2-0}{n} = \frac{2}{n}$, $x_i = 0 + i \Delta x = \frac{2i}{n}$, and $x_i^* = x_i$, the expression for the

area is $A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(5 + \frac{2i}{n}\right)^{10} \frac{2}{n}$. Note that the answer is not unique. We could use $y = x^{10}$

on $[5, 7]$ or, in general, $y = ((5 - n) + x)^{10}$ on $[n, n + 2]$.

12. $\Delta x = (4 - 1)/6 = \frac{1}{2}$, so the endpoints are 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, 3, $\frac{7}{2}$, and 4, and the midpoints are $\frac{5}{4}$, $\frac{7}{4}$, $\frac{9}{4}$, $\frac{11}{4}$, $\frac{13}{4}$, and $\frac{15}{4}$.

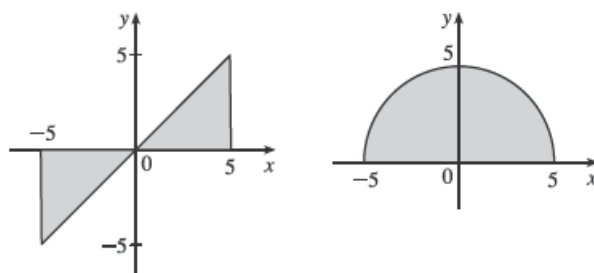
The Midpoint Rule gives

$$\begin{aligned} \int_1^4 \sqrt{x^3 + 1} dx &\approx \sum_{i=1}^6 f(\bar{x}_i) \Delta x \\ &= \frac{1}{2} \left(\sqrt{\left(\frac{5}{4}\right)^3 + 1} + \sqrt{\left(\frac{7}{4}\right)^3 + 1} + \sqrt{\left(\frac{9}{4}\right)^3 + 1} + \sqrt{\left(\frac{11}{4}\right)^3 + 1} + \sqrt{\left(\frac{13}{4}\right)^3 + 1} + \sqrt{\left(\frac{15}{4}\right)^3 + 1} \right) \\ &\approx 12.8516 \end{aligned}$$

24. Note that $\Delta x = \frac{2-0}{n} = \frac{2}{n}$ and $x_i = 0 + i \Delta x = \frac{2i}{n}$.

$$\begin{aligned} \int_0^2 (2x - x^3) dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{2i}{n}\right) \frac{2}{n} = \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left[2\left(\frac{2i}{n}\right) - \left(\frac{2i}{n}\right)^3 \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left[\frac{4i}{n} - \frac{8i^3}{n^3} \right] = \lim_{n \rightarrow \infty} \frac{2}{n} \left[\frac{4}{n} \sum_{i=1}^n i - \frac{8}{n^3} \sum_{i=1}^n i^3 \right] \\ &= \lim_{n \rightarrow \infty} \left\{ \frac{8}{n^2} \frac{n(n+1)}{2} - \frac{16}{n^4} \left[\frac{n(n+1)}{2} \right]^2 \right\} = \lim_{n \rightarrow \infty} \left[4 \frac{n+1}{n} - 4 \frac{(n+1)^2}{n^2} \right] \\ &= \lim_{n \rightarrow \infty} \left[4 \left(1 + \frac{1}{n} \right) - 4 \frac{n+1}{n} \frac{n+1}{n} \right] = \lim_{n \rightarrow \infty} \left[4 \left(1 + \frac{1}{n} \right) - 4 \left(1 + \frac{1}{n} \right) \left(1 + \frac{1}{n} \right) \right] \\ &= 4(1) - 4(1)(1) = 0 \end{aligned}$$

38. $\int_{-5}^5 (x - \sqrt{25 - x^2}) dx = \int_{-5}^5 x dx - \int_{-5}^5 \sqrt{25 - x^2} dx$. By symmetry, the value of the first integral is 0 since the shaded area above the x -axis equals the shaded area below the x -axis. The second integral can be interpreted as one half the area of a circle with radius 5; that is, $\frac{1}{2} \pi (5)^2 = \frac{25}{2} \pi$. Thus, the value of the original integral is $0 - \frac{25}{2} \pi = -\frac{25}{2} \pi$.



68. (a) Since $-|f(x)| \leq f(x) \leq |f(x)|$, it follows from Property 7 that

$$-\int_a^b |f(x)| dx \leq \int_a^b f(x) dx \leq \int_a^b |f(x)| dx \Rightarrow \left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx$$

Note that the definite integral is a real number, and so the following property applies: $-a \leq b \leq a \Rightarrow |b| \leq a$ for all real numbers b and nonnegative numbers a .

(b) $\left| \int_0^{2\pi} f(x) \sin 2x dx \right| \leq \int_0^{2\pi} |f(x) \sin 2x| dx$ [by part (a)] $= \int_0^{2\pi} |f(x)| |\sin 2x| dx \leq \int_0^{2\pi} |f(x)| dx$ by Property 7, since $|\sin 2x| \leq 1 \Rightarrow |f(x)| |\sin 2x| \leq |f(x)|$.