

$$38. g(x) = x^{1/3} - x^{-2/3} \Rightarrow g'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3} = \frac{1}{3}x^{-5/3}(x+2) = \frac{x+2}{3x^{5/3}}.$$

$g'(-2) = 0$ and $g'(0)$ does not exist, but 0 is not in the domain of g , so the only critical number is -2 .

$$52. f(x) = \frac{x}{x^2 - x + 1}, [0, 3].$$

$$f'(x) = \frac{(x^2 - x + 1) - x(2x - 1)}{(x^2 - x + 1)^2} = \frac{x^2 - x + 1 - 2x^2 + x}{(x^2 - x + 1)^2} = \frac{1 - x^2}{(x^2 - x + 1)^2} = \frac{(1+x)(1-x)}{(x^2 - x + 1)^2} = 0 \Leftrightarrow x = \pm 1,$$

but $x = -1$ is not in the given interval, $[0, 3]$. $f(0) = 0$, $f(1) = 1$, and $f(3) = \frac{3}{7}$. So $f(1) = 1$ is the absolute maximum value and $f(0) = 0$ is the absolute minimum value.

$$68. g(x) = 2 + (x - 5)^3 \Rightarrow g'(x) = 3(x - 5)^2 \Rightarrow g'(5) = 0, \text{ so } 5 \text{ is a critical number. But } g(5) = 2 \text{ and } g \text{ takes on values } > 2 \text{ and values } < 2 \text{ in any open interval containing } 5, \text{ so } g \text{ does not have a local maximum or minimum at } 5.$$

12.

$f(x) = \frac{1}{x}, [1, 3]$. f is continuous and differentiable on $(-\infty, 0) \cup (0, \infty)$, so f is continuous on $[1, 3]$ and differentiable

on $(1, 3)$. $f'(c) = \frac{f(b) - f(a)}{b - a} \Leftrightarrow -\frac{1}{c^2} = \frac{f(3) - f(1)}{3 - 1} = \frac{\frac{1}{3} - 1}{2} = -\frac{1}{3} \Leftrightarrow c^2 = 3 \Leftrightarrow c = \pm\sqrt{3}$, but only $\sqrt{3}$ is in $(1, 3)$.

28.

f satisfies the conditions for the Mean Value Theorem, so we use this theorem on the interval $[-b, b]$: $\frac{f(b) - f(-b)}{b - (-b)} = f'(c)$

for some $c \in (-b, b)$. But since f is odd, $f(-b) = -f(b)$. Substituting this into the above equation, we get

$$\frac{f(b) + f(b)}{2b} = f'(c) \Rightarrow \frac{f(b)}{b} = f'(c).$$

10.

$$(a) f(x) = 4x^3 + 3x^2 - 6x + 1 \Rightarrow f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x - 1)(x + 1). \text{ Thus,}$$

$f'(x) > 0 \Leftrightarrow x < -1 \text{ or } x > \frac{1}{2}$ and $f'(x) < 0 \Leftrightarrow -1 < x < \frac{1}{2}$. So f is increasing on $(-\infty, -1)$ and $(\frac{1}{2}, \infty)$ and f is decreasing on $(-1, \frac{1}{2})$.

(b) f changes from increasing to decreasing at $x = -1$ and from decreasing to increasing at $x = \frac{1}{2}$. Thus, $f(-1) = 6$ is a local maximum value and $f(\frac{1}{2}) = -\frac{3}{4}$ is a local minimum value.

(c) $f''(x) = 24x + 6 = 6(4x + 1)$. $f''(x) > 0 \Leftrightarrow x > -\frac{1}{4}$ and $f''(x) < 0 \Leftrightarrow x < -\frac{1}{4}$. Thus, f is concave upward on $(-\frac{1}{4}, \infty)$ and concave downward on $(-\infty, -\frac{1}{4})$. There is an inflection point at $(-\frac{1}{4}, f(-\frac{1}{4})) = (-\frac{1}{4}, \frac{21}{8})$.

14.

(a) $f(x) = \cos^2 x - 2 \sin x$, $0 \leq x \leq 2\pi$. $f'(x) = -2 \cos x \sin x - 2 \cos x = -2 \cos x (1 + \sin x)$. Note that

$$1 + \sin x \geq 0 \quad [\text{since } \sin x \geq -1], \text{ with equality } \Leftrightarrow \sin x = -1 \Leftrightarrow x = \frac{3\pi}{2} \quad [\text{since } 0 \leq x \leq 2\pi] \Rightarrow$$

$$\cos x = 0. \text{ Thus, } f'(x) > 0 \Leftrightarrow \cos x < 0 \Leftrightarrow \frac{\pi}{2} < x < \frac{3\pi}{2} \text{ and } f'(x) < 0 \Leftrightarrow \cos x > 0 \Leftrightarrow 0 < x < \frac{\pi}{2}$$

or $\frac{3\pi}{2} < x < 2\pi$. Thus, f is increasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$ and f is decreasing on $(0, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$.

(b) f changes from decreasing to increasing at $x = \frac{\pi}{2}$ and from increasing to decreasing at $x = \frac{3\pi}{2}$. Thus, $f(\frac{\pi}{2}) = -2$ is a

local minimum value and $f(\frac{3\pi}{2}) = 2$ is a local maximum value.

$$(c) f''(x) = 2 \sin x (1 + \sin x) - 2 \cos^2 x = 2 \sin x + 2 \sin^2 x - 2(1 - \sin^2 x)$$

$$= 4 \sin^2 x + 2 \sin x - 2 = 2(2 \sin x - 1)(\sin x + 1)$$

$$\text{so } f''(x) > 0 \Leftrightarrow \sin x > \frac{1}{2} \Leftrightarrow \frac{\pi}{6} < x < \frac{5\pi}{6}, \text{ and } f''(x) < 0 \Leftrightarrow \sin x < \frac{1}{2} \text{ and } \sin x \neq -1 \Leftrightarrow$$

$$0 < x < \frac{\pi}{6} \text{ or } \frac{5\pi}{6} < x < \frac{3\pi}{2} \text{ or } \frac{3\pi}{2} < x < 2\pi. \text{ Thus, } f \text{ is concave upward on } (\frac{\pi}{6}, \frac{5\pi}{6}) \text{ and concave downward on } (0, \frac{\pi}{6}),$$

$$(\frac{5\pi}{6}, \frac{3\pi}{2}), \text{ and } (\frac{3\pi}{2}, 2\pi). \text{ There are inflection points at } (\frac{\pi}{6}, -\frac{1}{4}) \text{ and } (\frac{5\pi}{6}, -\frac{1}{4}).$$

$$54. y = \frac{1+x}{1+x^2} \Rightarrow y' = \frac{(1+x^2)(1) - (1+x)(2x)}{(1+x^2)^2} = \frac{1-2x-x^2}{(1+x^2)^2} \Rightarrow$$

$$y'' = \frac{(1+x^2)^2(-2-2x) - (1-2x-x^2) \cdot 2(1+x^2)(2x)}{[(1+x^2)^2]^2} = \frac{2(1+x^2)[(1+x^2)(-1-x) - (1-2x-x^2)(2x)]}{(1+x^2)^4}$$

$$= \frac{2(-1-x-x^2-x^3-2x+4x^2+2x^3)}{(1+x^2)^3} = \frac{2(x^3+3x^2-3x-1)}{(1+x^2)^3} = \frac{2(x-1)(x^2+4x+1)}{(1+x^2)^3}$$

$$\text{So } y'' = 0 \Rightarrow x = 1, -2 \pm \sqrt{3}. \text{ Let } a = -2 - \sqrt{3}, b = -2 + \sqrt{3}, \text{ and } c = 1. \text{ We can show that } f(a) = \frac{1}{4}(1 - \sqrt{3}),$$

$$f(b) = \frac{1}{4}(1 + \sqrt{3}), \text{ and } f(c) = 1. \text{ To show that these three points of inflection lie on one straight line, we'll show that the}$$

slopes m_{ac} and m_{bc} are equal.

$$m_{ac} = \frac{f(c) - f(a)}{c - a} = \frac{1 - \frac{1}{4}(1 - \sqrt{3})}{1 - (-2 - \sqrt{3})} = \frac{\frac{3}{4} + \frac{1}{4}\sqrt{3}}{3 + \sqrt{3}} = \frac{1}{4}$$

$$m_{bc} = \frac{f(c) - f(b)}{c - b} = \frac{1 - \frac{1}{4}(1 + \sqrt{3})}{1 - (-2 + \sqrt{3})} = \frac{\frac{3}{4} - \frac{1}{4}\sqrt{3}}{3 - \sqrt{3}} = \frac{1}{4}$$