

16. Using Exercise 2.3.85(a), $f(x) = x^2 \sin x \tan x \Rightarrow$

$$\begin{aligned} f'(x) &= (x^2)' \sin x \tan x + x^2 (\sin x)' \tan x + x^2 \sin x (\tan x)' = 2x \sin x \tan x + x^2 \cos x \tan x + x^2 \sin x \sec^2 x \\ &= 2x \sin x \tan x + x^2 \sin x + x^2 \sin x \sec^2 x = x \sin x (2 \tan x + x + x \sec^2 x). \end{aligned}$$

34.

$$\begin{aligned} y = \frac{\cos x}{2 + \sin x} \Rightarrow y' &= \frac{(2 + \sin x)(-\sin x) - \cos x \cos x}{(2 + \sin x)^2} = \frac{-2 \sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2} = \frac{-2 \sin x - 1}{(2 + \sin x)^2} = 0 \text{ when} \\ -2 \sin x - 1 &= 0 \Leftrightarrow \sin x = -\frac{1}{2} \Leftrightarrow x = \frac{11\pi}{6} + 2\pi n \text{ or } x = \frac{7\pi}{6} + 2\pi n, n \text{ an integer. So } y = \frac{1}{\sqrt{3}} \text{ or } y = -\frac{1}{\sqrt{3}} \text{ and} \\ \text{the points on the curve with horizontal tangents are: } &\left(\frac{11\pi}{6} + 2\pi n, \frac{1}{\sqrt{3}}\right), \left(\frac{7\pi}{6} + 2\pi n, -\frac{1}{\sqrt{3}}\right), n \text{ an integer.} \end{aligned}$$

$$42. \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\sin \theta} = \lim_{\theta \rightarrow 0} \frac{\frac{\cos \theta - 1}{\theta}}{\frac{\sin \theta}{\theta}} = \frac{\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta}}{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}} = \frac{0}{1} = 0$$

52.

$$\begin{aligned} y = \sqrt{1+x^3} &= (1+x^3)^{1/2} \Rightarrow y' = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2 = \frac{3x^2}{2\sqrt{1+x^3}}. \text{ At } (2, 3), y' = \frac{3 \cdot 4}{2\sqrt{9}} = 2, \text{ and an equation of} \\ \text{the tangent line is } y - 3 &= 2(x - 2), \text{ or } y = 2x - 1. \end{aligned}$$

$$62. h(x) = \sqrt{4+3f(x)} \Rightarrow h'(x) = \frac{1}{2}(4+3f(x))^{-1/2} \cdot 3f'(x), \text{ so}$$

$$h'(1) = \frac{1}{2}(4+3f(1))^{-1/2} \cdot 3f'(1) = \frac{1}{2}(4+3 \cdot 7)^{-1/2} \cdot 3 \cdot 4 = \frac{6}{\sqrt{25}} = \frac{6}{5}$$

$$90. \text{ From Exercise 89, } \frac{d^2 y}{dx^2} = \frac{d^2 y}{du^2} \left(\frac{du}{dx}\right)^2 + \frac{dy}{du} \frac{d^2 u}{dx^2} \Rightarrow$$

$$\begin{aligned} \frac{d^3 y}{dx^3} &= \frac{d}{dx} \frac{d^2 y}{dx^2} = \frac{d}{dx} \left[\frac{d^2 y}{du^2} \left(\frac{du}{dx}\right)^2 \right] + \frac{d}{dx} \left[\frac{dy}{du} \frac{d^2 u}{dx^2} \right] \\ &= \left[\frac{d}{dx} \left(\frac{d^2 y}{du^2} \right) \right] \left(\frac{du}{dx}\right)^2 + \left[\frac{d}{dx} \left(\frac{du}{dx}\right)^2 \right] \frac{d^2 y}{du^2} + \left[\frac{d}{dx} \left(\frac{dy}{du}\right) \right] \frac{d^2 u}{dx^2} + \left[\frac{d}{dx} \left(\frac{d^2 u}{dx^2}\right) \right] \frac{dy}{du} \\ &= \left[\frac{d}{du} \left(\frac{d^2 y}{du^2} \right) \frac{du}{dx} \right] \left(\frac{du}{dx}\right)^2 + 2 \frac{du}{dx} \frac{d^2 u}{dx^2} \frac{d^2 y}{du^2} + \left[\frac{d}{du} \left(\frac{dy}{du}\right) \frac{du}{dx} \right] \left(\frac{d^2 u}{dx^2}\right) + \frac{d^3 u}{dx^3} \frac{dy}{du} \\ &= \frac{d^3 y}{du^3} \left(\frac{du}{dx}\right)^3 + 3 \frac{du}{dx} \frac{d^2 u}{dx^2} \frac{d^2 y}{du^2} + \frac{dy}{du} \frac{d^3 u}{dx^3} \end{aligned}$$

$$14. \frac{d}{dx} [y \sin(x^2)] = \frac{d}{dx} [x \sin(y^2)] \Rightarrow y \cos(x^2) \cdot 2x + \sin(x^2) \cdot y' = x \cos(y^2) \cdot 2y y' + \sin(y^2) \cdot 1 \Rightarrow$$

$$y' [\sin(x^2) - 2xy \cos(y^2)] = \sin(y^2) - 2xy \cos(x^2) \Rightarrow y' = \frac{\sin(y^2) - 2xy \cos(x^2)}{\sin(x^2) - 2xy \cos(y^2)}$$

$$30. x^{2/3} + y^{2/3} = 4 \Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0 \Rightarrow \frac{1}{\sqrt[3]{x}} + \frac{y'}{\sqrt[3]{y}} = 0 \Rightarrow y' = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}. \text{ When } x = -3\sqrt{3}$$

and $y = 1$, we have $y' = -\frac{1}{(-3\sqrt{3})^{1/3}} = -\frac{(-3\sqrt{3})^{2/3}}{-3\sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$, so an equation of the tangent line is

$$y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3}) \text{ or } y = \frac{1}{\sqrt{3}}x + 4.$$

40.

If $x = 1$ in $x^2 + xy + y^3 = 1$, then we get $1 + y + y^3 = 1 \Rightarrow y^3 + y = 0 \Rightarrow y(y^2 + 1) = 0 \Rightarrow y = 0$, so the point where $x = 1$ is $(1, 0)$. Differentiating implicitly with respect to x gives us $2x + xy' + y \cdot 1 + 3y^2 \cdot y' = 0$. Substituting 1 for x and 0 for y gives us $2 + y' + 0 + 0 = 0 \Rightarrow y' = -2$. Differentiating $2x + xy' + y + 3y^2y' = 0$ implicitly with respect to x gives us $2 + xy'' + y' \cdot 1 + y' + 3(y^2y'' + y' \cdot 2yy') = 0$. Now substitute 1 for x , 0 for y , and -2 for y' .

$2 + y'' + (-2) + (-2) + 3(0 + 0) = 0 \Rightarrow y'' = 2$. Differentiating $2 + xy'' + 2y' + 3y^2y'' + 6y(y')^2 = 0$ implicitly with respect to x gives us $xy''' + y'' \cdot 1 + 2y'' + 3(y^2y''' + y'' \cdot 2yy') + 6[y \cdot 2y'y'' + (y')^2y'] = 0$. Now substitute 1 for x , 0 for y , -2 for y' , and 2 for y'' . $y''' + 2 + 4 + 3(0 + 0) + 6[0 + (-8)] = 0 \Rightarrow y''' = -2 - 4 + 48 = 42$.

62.

$x^2 + 4y^2 = 5 \Rightarrow 2x + 4(2yy') = 0 \Rightarrow y' = -\frac{x}{4y}$. Now let h be the height of the lamp, and let (a, b) be the point of

tangency of the line passing through the points $(3, h)$ and $(-5, 0)$. This line has slope $(h - 0)/[3 - (-5)] = \frac{1}{8}h$. But the

slope of the tangent line through the point (a, b) can be expressed as $y' = -\frac{a}{4b}$, or as $\frac{b - 0}{a - (-5)} = \frac{b}{a + 5}$ [since the line

passes through $(-5, 0)$ and (a, b)], so $-\frac{a}{4b} = \frac{b}{a + 5} \Leftrightarrow 4b^2 = -a^2 - 5a \Leftrightarrow a^2 + 4b^2 = -5a$. But $a^2 + 4b^2 = 5$

[since (a, b) is on the ellipse], so $5 = -5a \Leftrightarrow a = -1$. Then $4b^2 = -a^2 - 5a = -1 - 5(-1) = 4 \Rightarrow b = 1$, since the

point is on the top half of the ellipse. So $\frac{h}{8} = \frac{b}{a + 5} = \frac{1}{-1 + 5} = \frac{1}{4} \Rightarrow h = 2$. So the lamp is located 2 units above the

x -axis.