24. $g'(t) = \lim_{h\to 0} \frac{g(t+h)-g(t)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{t}-\sqrt{t+h}}{\sqrt{t+h}\,\sqrt{t}}}{h} = \lim_{h\to 0} \left(\frac{\sqrt{t}-\sqrt{t+h}}{h\sqrt{t+h}\,\sqrt{t}} \cdot \frac{\sqrt{t}+\sqrt{t+h}}{\sqrt{t}+\sqrt{t+h}} \right)$

$= \lim_{h\to 0} \frac{t-(t+h)}{h\sqrt{t+h}\,\sqrt{t}\,(\sqrt{t}+\sqrt{t+h})} = \lim_{h\to 0} \frac{-h}{h\sqrt{t+h}\,\sqrt{t}\,(\sqrt{t}+\sqrt{t+h})} = \lim_{h\to 0} \frac{-1}{\sqrt{t+h}\,\sqrt{t}\,(\sqrt{t}+\sqrt{t+h})}$

$= \frac{-1}{\sqrt{t}\,\sqrt{t}\,(\sqrt{t}+\sqrt{t})} = \frac{-1}{t\,(2\sqrt{t})} = -\frac{1}{2t^{3/2}}$

Domain of g = domain of $g' = (0, \infty)$.

38. f is not differentiable at $x = -1$, because there is a discontinuity there, and at $x = 2$, because the graph has a corner there.

49.

(a) Note that we have factored $x - a$ as the difference of two cubes in the third step.

$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{x^{1/3}-a^{1/3}}{x-a} = \lim_{x\to a} \frac{x^{1/3}-a^{1/3}}{(x^{1/3}-a^{1/3})(x^{2/3}+x^{1/3}a^{1/3}+a^{2/3})}$

$= \lim_{x\to a} \frac{1}{x^{2/3}+x^{1/3}a^{1/3}+a^{2/3}} = \frac{1}{3a^{2/3}}$ or $\frac{1}{3}a^{-2/3}$

(b) $f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\sqrt[3]{h}-0}{h} = \lim_{h\to 0} \frac{1}{h^{2/3}}$. This function increases without bound, so the limit does not exist, and therefore $f'(0)$ does not exist.

(c) $\lim_{x\to 0} |f'(x)| = \lim_{x\to 0} \frac{1}{3x^{2/3}} = \infty$ and f is continuous at $x = 0$ (root function), so f has a vertical tangent at $x = 0$.

32. $y = \frac{x+1}{x^3+x-2} \quad \overset{\text{QR}}{\Rightarrow}$

$y' = \frac{(x^3+x-2)(1)-(x+1)(3x^2+1)}{(x^3+x-2)^2} = \frac{x^3+x-2-3x^3-3x^2-x-1}{(x^3+x-2)^2} = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$

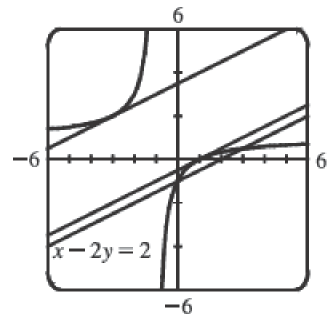
or $-\frac{2x^3+3x^2+3}{(x-1)^2(x^2+x+2)^2}$

70. $\frac{d}{dx}\left[\frac{h(x)}{x}\right] = \frac{xh'(x)-h(x)\cdot 1}{x^2} \quad \Rightarrow \quad \frac{d}{dx}\left[\frac{h(x)}{x}\right]_{x=2} = \frac{2h'(2)-h(2)}{2^2} = \frac{2(-3)-(4)}{4} = \frac{-10}{4} = -2.5$

76. $f(x) = x^3+3x^2+x+3$ has a horizontal tangent when $f'(x) = 3x^2+6x+1 = 0 \quad \Leftrightarrow$

$x = \frac{-6 \pm \sqrt{36-12}}{6} = -1 \pm \frac{1}{3}\sqrt{6}$.

80. $y = \dfrac{x-1}{x+1} \quad\Rightarrow\quad y' = \dfrac{(x+1)(1)-(x-1)(1)}{(x+1)^2} = \dfrac{2}{(x+1)^2}$. If the tangent intersects the curve when $x = a$, then its slope is $2/(a+1)^2$. But if the tangent is parallel to $x - 2y = 2$, that is, $y = \frac{1}{2}x - 1$, then its slope is $\frac{1}{2}$. Thus, $\dfrac{2}{(a+1)^2} = \dfrac{1}{2} \quad\Rightarrow$ $(a+1)^2 = 4 \quad\Rightarrow\quad a+1 = \pm 2 \quad\Rightarrow\quad a = 1$ or -3. When $a = 1$, $y = 0$ and the equation of the tangent is $y - 0 = \frac{1}{2}(x-1)$ or $y = \frac{1}{2}x - \frac{1}{2}$. When $a = -3$, $y = 2$ and the equation of the tangent is $y - 2 = \frac{1}{2}(x+3)$ or $y = \frac{1}{2}x + \frac{7}{2}$.



100.
f is clearly differentiable for $x < 2$ and for $x > 2$. For $x < 2$, $f'(x) = 2x$, so $f'_-(2) = 4$. For $x > 2$, $f'(x) = m$, so $f'_+(2) = m$. For f to be differentiable at $x = 2$, we need $4 = f'_-(2) = f'_+(2) = m$. So $f(x) = 4x + b$. We must also have continuity at $x = 2$, so $4 = f(2) = \lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (4x + b) = 8 + b$. Hence, $b = -4$.