24. $f(x) = \dfrac{x^3-8}{x^2-4} = \dfrac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \dfrac{x^2+2x+4}{x+2}$ for $x \neq 2$. Since $\lim_{x\to 2} f(x) = \dfrac{4+4+4}{2+2} = 3$, define $f(2) = 3$.

Then f is continuous at 2.

30.

By Theorem 7, the trigonometric function $\tan x$ is continuous on its domain, $\left\{x \mid x \neq \frac{\pi}{2} + \pi n\right\}$. By Theorems 5(a), 7, and 9, the composite function $\sqrt{4-x^2}$ is continuous on its domain $[-2, 2]$. By part 5 of Theorem 4, $B(x) = \dfrac{\tan x}{\sqrt{4-x^2}}$ is continuous on its domain, $(-2, -\pi/2) \cup (-\pi/2, \pi/2) \cup (\pi/2, 2)$.

46. $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$

At $x = 2$: $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \dfrac{x^2-4}{x-2} = \lim_{x\to 2^-} \dfrac{(x+2)(x-2)}{x-2} = \lim_{x\to 2^-}(x+2) = 2+2 = 4$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(ax^2 - bx + 3) = 4a - 2b + 3$

We must have $4a - 2b + 3 = 4$, or $\mathbf{4a - 2b = 1}$ **(1)**.

At $x = 3$: $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(ax^2 - bx + 3) = 9a - 3b + 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(2x - a + b) = 6 - a + b$

We must have $9a - 3b + 3 = 6 - a + b$, or $\mathbf{10a - 4b = 3}$ **(2)**.

Now solve the system of equations by adding -2 times equation **(1)** to equation **(2)**.

$$\begin{array}{rl} -8a + 4b = & -2 \\ 10a - 4b = & 3 \\ \hline 2a \quad = & 1 \end{array}$$

So $a = \frac{1}{2}$. Substituting $\frac{1}{2}$ for a in **(1)** gives us $-2b = -1$, so $b = \frac{1}{2}$ as well. Thus, for f to be continuous on $(-\infty, \infty)$, $a = b = \frac{1}{2}$.

66. $\dfrac{a}{x^3+2x^2-1} + \dfrac{b}{x^3+x-2} = 0 \quad \Rightarrow \quad a(x^3+x-2) + b(x^3+2x^2-1) = 0$. Let $p(x)$ denote the left side of the last equation. Since p is continuous on $[-1, 1]$, $p(-1) = -4a < 0$, and $p(1) = 2b > 0$, there exists a c in $(-1, 1)$ such that $p(c) = 0$ by the Intermediate Value Theorem. Note that the only root of either denominator that is in $(-1, 1)$ is $(-1+\sqrt{5})/2 = r$, but $p(r) = (3\sqrt{5} - 9)a/2 \neq 0$. Thus, c is not a root of either denominator, so $p(c) = 0 \quad \Rightarrow$ $x = c$ is a root of the given equation.

12.

(a) **Runner A** runs the entire 100-meter race at the same velocity since the slope of the position function is constant.

Runner B starts the race at a slower velocity than runner A, but finishes the race at a faster velocity.

(b) The distance between the runners is the greatest at the time when the largest vertical line segment fits between the two graphs—this appears to be somewhere between 9 and 10 seconds.

(c) The runners had the same velocity when the slopes of their respective position functions are equal—this also appears to be at about 9.5 s. Note that the answers for parts (b) and (c) must be the same for these graphs because as soon as the velocity for runner B overtakes the velocity for runner A, the distance between the runners starts to decrease.

14.

(a) Let $H(t) = 10t - 1.86t^2$.

$$v(1) = \lim_{h\to 0} \frac{H(1+h) - H(1)}{h} = \lim_{h\to 0} \frac{\left[10(1+h) - 1.86(1+h)^2\right] - (10 - 1.86)}{h}$$

$$= \lim_{h\to 0} \frac{10 + 10h - 1.86(1 + 2h + h^2) - 10 + 1.86}{h}$$

$$= \lim_{h\to 0} \frac{10 + 10h - 1.86 - 3.72h - 1.86h^2 - 10 + 1.86}{h}$$

$$= \lim_{h\to 0} \frac{6.28h - 1.86h^2}{h} = \lim_{h\to 0}(6.28 - 1.86h) = 6.28$$

The velocity of the rock after one second is 6.28 m/s.

(b) $$v(a) = \lim_{h\to 0} \frac{H(a+h) - H(a)}{h} = \lim_{h\to 0} \frac{\left[10(a+h) - 1.86(a+h)^2\right] - (10a - 1.86a^2)}{h}$$

$$= \lim_{h\to 0} \frac{10a + 10h - 1.86(a^2 + 2ah + h^2) - 10a + 1.86a^2}{h}$$

$$= \lim_{h\to 0} \frac{10a + 10h - 1.86a^2 - 3.72ah - 1.86h^2 - 10a + 1.86a^2}{h} = \lim_{h\to 0} \frac{10h - 3.72ah - 1.86h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(10 - 3.72a - 1.86h)}{h} = \lim_{h\to 0}(10 - 3.72a - 1.86h) = 10 - 3.72a$$

The velocity of the rock when $t = a$ is $(10 - 3.72a)$ m/s.

(c) The rock will hit the surface when $H = 0 \quad\Leftrightarrow\quad 10t - 1.86t^2 = 0 \quad\Leftrightarrow\quad t(10 - 1.86t) = 0 \quad\Leftrightarrow\quad t = 0$ or $1.86t = 10$.

The rock hits the surface when $t = 10/1.86 \approx 5.4$ s.

(d) The velocity of the rock when it hits the surface is $v\left(\frac{10}{1.86}\right) = 10 - 3.72\left(\frac{10}{1.86}\right) = 10 - 20 = -10$ m/s.

32. Use (4) with $f(x) = \dfrac{4}{\sqrt{1-x}}$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{4}{\sqrt{1-(a+h)}} - \dfrac{4}{\sqrt{1-a}}}{h}$$

$$= 4 \lim_{h\to 0} \frac{\dfrac{\sqrt{1-a}-\sqrt{1-a-h}}{\sqrt{1-a-h}\,\sqrt{1-a}}}{h} = 4 \lim_{h\to 0} \frac{\sqrt{1-a}-\sqrt{1-a-h}}{h\sqrt{1-a-h}\,\sqrt{1-a}}$$

$$= 4 \lim_{h\to 0} \frac{\sqrt{1-a}-\sqrt{1-a-h}}{h\sqrt{1-a-h}\,\sqrt{1-a}} \cdot \frac{\sqrt{1-a}+\sqrt{1-a-h}}{\sqrt{1-a}+\sqrt{1-a-h}} = 4 \lim_{h\to 0} \frac{(\sqrt{1-a})^2-(\sqrt{1-a-h})^2}{h\sqrt{1-a-h}\,\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a-h})}$$

$$= 4 \lim_{h\to 0} \frac{(1-a)-(1-a-h)}{h\sqrt{1-a-h}\,\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a-h})} = 4 \lim_{h\to 0} \frac{h}{h\sqrt{1-a-h}\,\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a-h})}$$

$$= 4 \lim_{h\to 0} \frac{1}{\sqrt{1-a-h}\,\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a-h})} = 4 \cdot \frac{1}{\sqrt{1-a}\,\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a})}$$

$$= \frac{4}{(1-a)(2\sqrt{1-a})} = \frac{2}{(1-a)^1(1-a)^{1/2}} = \frac{2}{(1-a)^{3/2}}$$

46. $\Delta V = V(t+h) - V(t) = 100{,}000\left(1-\dfrac{t+h}{60}\right)^2 - 100{,}000\left(1-\dfrac{t}{60}\right)^2$

$$= 100{,}000\left[\left(1-\frac{t+h}{30}+\frac{(t+h)^2}{3600}\right)-\left(1-\frac{t}{30}+\frac{t^2}{3600}\right)\right] = 100{,}000\left(-\frac{h}{30}+\frac{2th}{3600}+\frac{h^2}{3600}\right)$$

$$= \frac{100{,}000}{3600}h\,(-120+2t+h) = \frac{250}{9}h\,(-120+2t+h)$$

Dividing ΔV by h and then letting $h \to 0$, we see that the instantaneous rate of change is $\frac{500}{9}\,(t-60)$ gal/min.

t	Flow rate (gal/min)	Water remaining $V(t)$ (gal)
0	$-3333.\overline{3}$	$100,000$
10	$-2777.\overline{7}$	$69,444.\overline{4}$
20	$-2222.\overline{2}$	$44,444.\overline{4}$
30	$-1666.\overline{6}$	$25,000$
40	$-1111.\overline{1}$	$11,111.\overline{1}$
50	$-555.\overline{5}$	$2,777.\overline{7}$
60	0	0

The magnitude of the flow rate is greatest at the beginning and gradually decreases to 0.