

6. (a) $y = y(t) = 10t - 1.86t^2$. At $t = 1$, $y = 10(1) - 1.86(1)^2 = 8.14$. The average velocity between times 1 and $1 + h$ is

$$v_{\text{ave}} = \frac{y(1+h) - y(1)}{(1+h) - 1} = \frac{[10(1+h) - 1.86(1+h)^2] - 8.14}{h} = \frac{6.28h - 1.86h^2}{h} = 6.28 - 1.86h, \text{ if } h \neq 0.$$

(i) $[1, 2]: h = 1, v_{\text{ave}} = 4.42 \text{ m/s}$

(ii) $[1, 1.5]: h = 0.5, v_{\text{ave}} = 5.35 \text{ m/s}$

(iii) $[1, 1.1]: h = 0.1, v_{\text{ave}} = 6.094 \text{ m/s}$

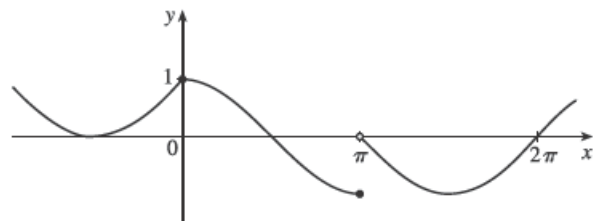
(iv) $[1, 1.01]: h = 0.01, v_{\text{ave}} = 6.2614 \text{ m/s}$

(v) $[1, 1.001]: h = 0.001, v_{\text{ave}} = 6.27814 \text{ m/s}$

(b) The instantaneous velocity when $t = 1$ (h approaches 0) is 6.28 m/s .

12. From the graph of

$$f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leq x \leq \pi, \\ \sin x & \text{if } x > \pi \end{cases}$$



we see that $\lim_{x \rightarrow a} f(x)$ exists for all a except $a = \pi$. Notice that the right and left limits are different at $a = \pi$.

20. For $f(x) = \frac{x^2 - 2x}{x^2 - x - 2}$:

x	$f(x)$
0	0
-0.5	-1
-0.9	-9
-0.95	-19
-0.99	-99
-0.999	-999

x	$f(x)$
-2	2
-1.5	3
-1.1	11
-1.01	101
-1.001	1001

It appears that $\lim_{x \rightarrow -1} \frac{x^2 - 2x}{x^2 - x - 2}$ does not exist since

$f(x) \rightarrow \infty$ as $x \rightarrow -1^-$ and $f(x) \rightarrow -\infty$ as $x \rightarrow -1^+$.

34.

$\lim_{x \rightarrow \pi^-} \cot x = \lim_{x \rightarrow \pi^-} \frac{\cos x}{\sin x} = -\infty$ since the numerator is negative and the denominator approaches 0 through positive values as $x \rightarrow \pi^-$.

$$\begin{aligned}
8. \lim_{t \rightarrow 2} \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right)^2 &= \left(\lim_{t \rightarrow 2} \frac{t^2 - 2}{t^3 - 3t + 5} \right)^2 && \text{[Limit Law 6]} \\
&= \left(\frac{\lim_{t \rightarrow 2} (t^2 - 2)}{\lim_{t \rightarrow 2} (t^3 - 3t + 5)} \right)^2 && [5] \\
&= \left(\frac{\lim_{t \rightarrow 2} t^2 - \lim_{t \rightarrow 2} 2}{\lim_{t \rightarrow 2} t^3 - 3 \lim_{t \rightarrow 2} t + \lim_{t \rightarrow 2} 5} \right)^2 && [1, 2, \text{ and } 3] \\
&= \left(\frac{4 - 2}{8 - 3(2) + 5} \right)^2 && [9, 7, \text{ and } 8] \\
&= \left(\frac{2}{7} \right)^2 = \frac{4}{49}
\end{aligned}$$

10.

(a) The left-hand side of the equation is not defined for $x = 2$, but the right-hand side is.

(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \rightarrow 2$, just as in Example 3. Remember that in finding $\lim_{x \rightarrow a} f(x)$, we never consider $x = a$.

$$\begin{aligned}
30. \lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4} &= \lim_{x \rightarrow -4} \frac{(\sqrt{x^2 + 9} - 5)(\sqrt{x^2 + 9} + 5)}{(x + 4)(\sqrt{x^2 + 9} + 5)} = \lim_{x \rightarrow -4} \frac{(x^2 + 9) - 25}{(x + 4)(\sqrt{x^2 + 9} + 5)} \\
&= \lim_{x \rightarrow -4} \frac{x^2 - 16}{(x + 4)(\sqrt{x^2 + 9} + 5)} = \lim_{x \rightarrow -4} \frac{(x + 4)(x - 4)}{(x + 4)(\sqrt{x^2 + 9} + 5)} \\
&= \lim_{x \rightarrow -4} \frac{x - 4}{\sqrt{x^2 + 9} + 5} = \frac{-4 - 4}{\sqrt{16 + 9} + 5} = \frac{-8}{5 + 5} = -\frac{4}{5}
\end{aligned}$$

$$58. (a) \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left[\frac{f(x)}{x^2} \cdot x^2 \right] = \lim_{x \rightarrow 0} \frac{f(x)}{x^2} \cdot \lim_{x \rightarrow 0} x^2 = 5 \cdot 0 = 0$$

$$(b) \lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \left[\frac{f(x)}{x^2} \cdot x \right] = \lim_{x \rightarrow 0} \frac{f(x)}{x^2} \cdot \lim_{x \rightarrow 0} x = 5 \cdot 0 = 0$$